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NAVAL POSTGRADUATE SCHOOL
Monterey, California

A SINGLE-COMMODITY MINE TRANSSHIPMENT PROBLEM

by

Tammy L. Glaser

September 1991

Thesis Advisor R. Kevin Wood

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The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command, on schedules for naval mine transshipment. This thesis develops and implements a model, Scheduler for Mine Transshipment (SUMIT), that generates optimal schedules for transporting mines of a single type to suitable staging sites and for laying mines in mine fields. The model considers the number of available air, land and sea assets such as military aircraft, trucks, submarines and ships in finding optimal schedules for mine transshipment. SUMIT is designed to solve problems for scenarios in a limited region of the world that last several days and is based on interconnected, time-expanded mine and mode networks. SUMIT is written in the General Algebraic Modeling System (GAMS) and is a mixed integer linear program in which all integer variables are binary. Ten realistic test problems are solved to demonstrate the viability of SUMIT and to compare the relative efficiencies of two model variants. One variant is on average 87% faster than the other.

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A Single-Commodity
Mine Transshipment Problem

by

Tammy L. Glaser
Lieutenant, United States Navy
B.S., United States Naval Academy, 1985

Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command, on schedules for naval mine transshipment. This thesis develops and implements a model, Scheduler for Mine Transshipment (SUMIT), that generates optimal schedules for transporting mines of a single type to suitable staging sites and for laying mines in mine fields. The model considers the number of available air, land and sea assets such as military aircraft, trucks, submarines and ships in finding optimal schedules for mine transshipment. SUMIT is designed to solve problems for scenarios in a limited region of the world that last several days and is based on interconnected, time-expanded mine and mode networks. SUMIT is written in the General Algebraic Modeling System (GAMS) and is a mixed integer linear program in which all integer variables are binary. Ten realistic test problems are solved to demonstrate the viability of SUMIT and to compare the relative efficiencies of two model variants. One variant is on average 87% faster than the other.

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THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

TABLE OF CONTENTS

I. INTRODUCTION 1
  A. PURPOSE 1
  B. BACKGROUND 1
    1. Node Description 2
    2. Mode Description 3
    3. Arc Description 4
  C. SCOPE 4
  D. OUTLINE 5

II. MODEL FORMULATION 6
  A. GENERAL DESCRIPTION 6
  B. ASSUMPTIONS 8
  C. FORMULATION 10
    1. Indices 10
    2. Data 12
    3. Decision Variables 14
    4. Model A 16
    5. Comments about Model A 17
    6. Model B 18
    7. Comments about Model B 19
    8. Detailed Data Description for Both Models 19
    9. Network Generation Rules for Both Models 26
    10. Generation of Incompatible Arc Groups 27
    11. Flow Existence Rules 30

III. COMPUTATIONAL RESULTS 32
  A. COMPUTER IMPLEMENTATION OF PROPOSED MODELS 32
  B. COMPARISON OF MODELS 32
  C. GAMS IMPLEMENTATION OF SUMIT 35
  D. REDUCTION OF MINE FLOW AND INVENTORY IN BOTH MODELS 36

IV. CONCLUSIONS ... 39

APPENDIX A. SAMPLE PROGRAM, FIRST RUN ... 41

APPENDIX B. SAMPLE OUTPUT, FIRST RUN ... 58

APPENDIX C. SAMPLE PROGRAM, SECOND RUN ... 62

APPENDIX D. SAMPLE OUTPUT, SECOND RUN ... 68

APPENDIX E. GAMS IMPLEMENTATION OF SUMIT ... 71
  A. SUMIT OUTPUT ... 71
  B. END TIME AND TIME PERIOD LENGTH ... 71
  C. OBTAINING NEARLY OPTIMAL SOLUTIONS ... 72

LIST OF REFERENCES ... 73

INITIAL DISTRIBUTION LIST ... 75

LIST OF TABLES

Table 1. INFORMATION ABOUT SCENARIOS FOR TEN TEST PROBLEMS 33
Table 2. RESULTS OF TEN TEST RUNS TO COMPARE BOTH MODELS . . 34
Table 3. MINE FLOW REDUCTION 37
Table 4. MINE INVENTORY REDUCTION 38

LIST OF FIGURES

Figure 1. Example of a Mine Network 7
Figure 2. Example of a Mode Network 8
Figure 3. Example of a Time-Expanded Mine Network 9
Figure 4. Example of a MOMAG-Supply Node Split in a Mine Network 10
Figure 5. Relationship Between Node Function and Dimension Subsets 13
Figure 6. Relationship Between Node Function and Arc Position Subsets 14
Figure 7. Relationship Between Mode Function and Dimension Subsets 15
Figure 8. Relationship between Arcs from the Same Node to Different Destinations 28
Figure 9. Illustration of Step 1 of Generating Incompatible Arc Groups 29
Figure 10. Illustration of Step 2 of Generating Incompatible Arc Groups 30
Figure 11. Illustration of the Final Step in Generating Incompatible Arc Groups . 31

I. INTRODUCTION

A. PURPOSE

The Fleet Commanders-in-Chief often request recommendations from Commander, Mine Warfare Command (CMWC), who supports mining operations executed by the U.S. Navy, on schedules for naval mine transshipment in the event of war or for minelaying exercises. Schedules for transporting mines to suitable staging sites and for laying mines in mine fields must be generated to meet the needs of mine warfare, given the number of available air, land, and sea assets (such as military aircraft, trucks, submarines, and ships). Only a limited number of military assets are capable of carrying mines due to special equipment requirements. Furthermore, since most mine transshipment and delivery assets are not solely dedicated to mine warfare, a mine transshipment scheduler must take into account both transshipment and delivery times and mine field and resource priorities. The current methods of planning mine transshipment schedules are manual, with no utilization of computerized optimization algorithms. Recognizing the need to computerize data and algorithms for many mine warfare problems, CMWC has initiated the installation of a computer system, the Geo-Operational Planning and Assessment System (GOPAS), to automate and optimize several aspects of mining operations [Ref. 1].

The purpose of this thesis is to build a computer-based mine transshipment model that can accept data stored by GOPAS and is automatic with respect to formulation and execution. While this thesis focuses on the rapid deployment of mines in regional real-time or exercise scenarios that take two to three days to complete, recommendations are also made for approaching problems of longer duration, such as a few months. The solutions of the Scheduler for Mine Transshipment (SUMIT) model can be used to recommend not only feasible schedules for mine transshipment but, in many cases, optimal (or close to optimal) schedules. While SUMIT recognizes time as a critical factor, its consideration of mine field and resource priorities allows the user to account for intangible aspects of the scenario by including both time and priority in the measure of effectiveness (MOE) used in the model.

B. BACKGROUND

The mine transshipment problem is modeled, using two networks. Nodes in the network represent supply, transshipment, and demand sites. In the network for mine flow, arcs represent possible transportation routes between nodes in which modes, such as trucks, aircraft, ships, and submarines, carry mines. The network for mode flow is similar to the mine network, except that additional return arcs are needed to represent the return trips of modes back to their home base.

1. Node Description

Nodes can be separated into three broad categories: supply, transshipment and demand. The U.S. Navy has 13 Mobile Mine Assembly Groups (MOMAGs) located throughout the world which store mines and function as supply nodes. These MOMAGs are typically collocated with large U.S. Navy and U.S. Air Force bases which have resources, such as aircraft capable of transporting and/or deploying mines. The mines at the MOMAGs are maintained at six levels of readiness, from disassembled to fully-assembled (ready-to-deploy). If mines are disassembled, MOMAGs have the facilities to prepare mines for delivery. The rate of assembling mines in preparation for deployment is called "build rate" and is measured in mines per hour. Some MOMAGs have the capability of setting up dual parallel assembly lines, thereby doubling the build rate. The U.S. Navy has an inventory of approximately 16 types of underwater mines and the build rate is known for each mine type [Ref. 2].

To decrease delivery times, fully-assembled mines are also pre-staged at locations other than the MOMAGs. They can be stored at bases that have mine delivery air modes, on aircraft carriers and surface ships to be deployed via ship-based air modes, or on submarines to be deployed by the submarine itself. Currently, surface ships do not have the capability to deploy mines but a new class of mine-laying ships, called High-Volume Mine Layers (HVMLs), are planned to fill this gap.

Transshipment nodes are typically U.S. Navy and U.S. Air Force bases with aircraft capable of supporting mine deployment. In this thesis, a military base that (1) has no pre-staged fully-assembled mines in inventory and (2) is not collocated with a MOMAG, is categorized as a transshipment node.

CMWC has developed mine field plans for a variety of scenarios that could occur throughout the world. In fact, the execution of all mine field plans simultaneously would deplete the entire supply of mines. Because the potential demand is much greater than the supply, all mine field plans are prioritized according to the importance of their objective. Approximately 90 percent of the time, mine field plans require only one type of mine and, therefore, generate only single-commodity scheduling problems.

Node data that can be obtained from GOPAS to support the proposed model SUMIT include the number of each mine type stored at every MOMAG at each level of readiness, the number of each mine type pre-staged at given bases, the number of each mine type demanded at every mine field, the build rate of every MOMAG for each mine type, the capability for a double assembly line, the priority of the mine field, and the type of node (land or sea).

2. Mode Description

Modes can be separated into two broad categories: transportation modes and delivery modes. Modes that cannot deploy mines in a mine field are categorized as transportation modes. These modes include land modes, such as trucks, and air modes, such as U.S. Air Force C-141 and C-130 cargo aircraft. Delivery modes can deploy mines and include air modes, such as U.S. Navy and U.S. Marine Corps shore-based and ship-based aircraft and U.S. Air Force B-52 bombers, and sea modes, such as submarines. Due to weight and size constraints, most air modes and all land modes can only carry one mine type per trip. Sea modes, which will include HVMLs in the future, are able to carry more than one mine type aboard. Because modes have multiple capabilities, they may also be critical to the success of other missions and can be prioritized according to the scenario. For example, if Anti-Submarine Warfare (ASW) plays a vital role in the scenario, a higher priority should be placed on non-ASW modes to deploy the mines.

Since the mine transshipment problem includes mobile supply sites (aircraft carriers, ships, and submarines), differences arise in how mobile supply sites are incorporated into the network. For this thesis, *resources normally classified as modes are treated as nodes if they function as supply nodes and can only store mines.* For example, aircraft carriers and surface ships, which store mines but are not capable of laying them, are classified as fixed nodes in the model. The aircraft stationed aboard the vessel function as modes that lay mines. To maintain consistency in the structure of the network, submarines and HVMLs, which are able to deploy mines, are treated as fixed nodes that have dummy modes stationed aboard to deliver mines. In addition, because aircraft carriers and ships may not be located at the on-station point for launch of aircraft at the beginning of the problem, SUMIT allows the transit of aircraft carriers and ships at the beginning and end of the problem. This exception is not required for submarines which travel directly to the mine fields.

Mode data supported by GOPAS include category (transshipment or delivery); dimension (land, air or sea); speed (nautical miles per hour); capacity for each mine type (mines per unit mode); time to load mines (hours); time to unload or deliver mines (hours); total time to refuel, change crews, and conduct routine maintenance or repairs to prepare modes for their next trip (hours); maximum range (nautical miles); the number of each mode available at each supply and transshipment node; and mode priority.

3. Arc Description

Because the flow of mines between nodes in the network is very structured, the only arc data that require support from GOPAS is the distance between nodes (nautical miles). Arc capacity (mines per trip) and length (time periods) can be calculated from the node data. Based on the data input by the user, SUMIT only forms arcs for mine networks that are directed from supply nodes to supply nodes, supply nodes to transshipment nodes, supply nodes to demand nodes, and transshipment nodes to demand nodes. SUMIT generates the same arcs for mode networks, except the return arcs are also included and are directed back to the home base. In Chapter II, the section entitled "Network Generation Rules" explains in detail other rules used to generate the networks.

C. SCOPE

Other researchers have studied the multi-commodity transshipment problem for mine warfare and for other military applications. Wingate and Zakary [Ref. 3] proposed a continuous variable model for multi-commodity transshipment problems, that could be applied to mine transshipment. However, their model was too general and did not address characteristics unique to the mine transshipment problem, such as the existence of modes (i.e., aircraft carriers and submarines) that also function as supply nodes.

Collier, Lally, and Puntenney [Refs. 4, 5 , 6] developed continuous variable models for military deployment problems from the U.S. Transportation Command (TRANSCOM) using sea and air assets. The mine transshipment problem is smaller with potential supply sites limited to the 13 MOMAGs plus the bases and sea assets at which mines are pre-staged. The proposed model SUMIT is designed for regional problems spanning a time window of two to three days in which the total number of nodes can be about ten, while the TRANSCOM model developed by Puntenney covers movement requests among as many as 22 ports, planning general schedules lasting up to three months. Due to the importance of time in a short-duration mine transshipment problem, sea assets are limited to U.S. Navy vessels, located near the mine fields at the beginning of the problem, that either have the capability to deploy mines or have air assets aboard that are able to lay mines, while sea assets play a prominent role in shipping material in the TRANSCOM problem. Since the mine transshipment problem requires mines to be deployed as soon as possible, it does not include opportune delivery times, which were used to generate costs for the objective function in the TRANSCOM problem. Finally, data concerning limits on the mine loading and unloading capacities at non-MOMAG supply sites and restrictions on the number of aircraft allowed at transshipment sites at one time is not available in GOPAS.

The scope of this thesis could encompass a multi-commodity transshipment model that optimizes schedules for deploying different mine types in a global scenario. Multi-commodity problems usually involve mixed-integer programs that use extensive amounts of computer resources to find optimal solutions for large networks. Because only one mine type can be transported by a mode for each trip, the complexity of the mine transshipment problem increases. Since very little preliminary groundwork has been laid which specifically addresses schedule optimization for the mine transshipment problem, the scope of this thesis has been narrowed to a single-commodity approach. Because this thesis focuses on regional scenarios that require mine deployment to a couple of mine fields over a period of several days, the use of a single-commodity model is justified by the fact that (1) most mine fields require only one type of mine and (2) the majority of MOMAGs do not supply all 16 types of mines. Furthermore, the transshipment of mines over long distances may take more time than is allotted for the scenario, which then restricts mine supply to sites in the same region of the world as the mine fields. Thus, a global problem could conceivably be divided into several regional subproblems, which could be solved separately, if nodes contained in one regional subproblem were not contained in the other regional subproblems.

D. OUTLINE

This thesis is divided into four chapters. Chapter I is the introduction. Chapter II proposes and describes in detail two model formulations. Chapter III discusses the results of ten test problems that compare the size and speed of the two model formulations and recommends procedures for executing SUMIT in the General Algebraic Modeling System (GAMS) [Ref. 7], a software package. Chapter IV contains conclusions about the methodology described in this thesis, discusses the weaknesses of SUMIT, and recommends future enhancements of SUMIT.

II. MODEL FORMULATION

A. GENERAL DESCRIPTION

Both proposed mine transshipment models contain two separate networks: one for the mines and another for the modes. The mine network is represented by a set of balance equations that controls the flow of mines through all nodes. Two types of variables are contained in the balance equations for the mine network: one variable type represents the shipment of mines from origin nodes to destination nodes via modes and the other represents the inventory of mines at nodes. The mode network is represented by another set of balance equations that controls the flow of modes to and from all nodes. The mine and mode networks are linked by a set of equations that relate the flow variables in the two sets of balance equations. In the linking equations, sending mines via an arc contained in the mine network, forces flow on the corresponding arc in the mode network. In other words, mines cannot flow through the mine network unless there is a mode to carry them. Furthermore, the number of mines flowing through the network is limited by the capacities of the associated modes.

The mine network is directed, meaning that arcs are ordered pairs of nodes. All paths must start at supply nodes, possibly flow through other supply nodes and transshipment nodes, and must arrive at demand nodes. Circuits, which are paths that start and end at the same node, are limited to transfers between supply nodes. Figure 1 illustrates a mine network, where s_i, t_i, and d_i respectively represent supply, transshipment, and demand at node i. Notice that the path from s_2 to t_1 contains two arcs, which indicates that two different types of modes are available to transport the mines. [Ref. 8]

The structure of the mode network is an expansion of the mine network, which has additional arcs returning empty modes back to their origin node. The return arcs are critical because they prevent arcs that represent the same mode leaving at a different time period from being used before the mode has returned from its previous run. Figure 2 depicts a mode network, associated with the mine network of Figure 2.

Time-expansion of nodes in the two networks and in the linking equations is required to allow modes to make multiple sequential trips in either transporting or laying mines. Different arc lengths (which are measured in time periods) also make time-expansion desirable. In essence, time-expansion expands every node over several periods

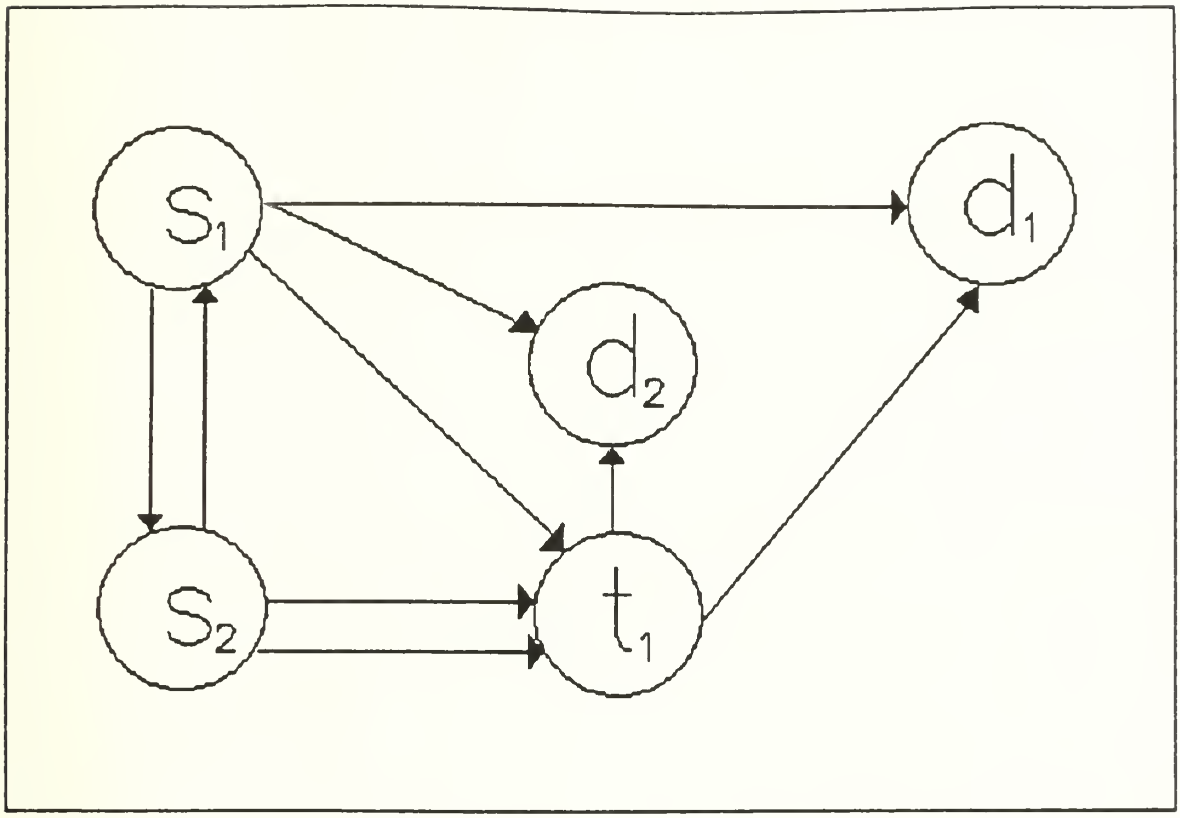


Figure 1. Example of a Mine Network

of time. The arcs of the time-expanded network connect earlier nodes to later nodes. Figure 3 illustrates a very simple time-expansion of a mine network.

To take into account the build rate of disassembled mines, a MOMAG-supply node is split into two nodes separated by an arc. The MOMAG node (the origin node), which has no arcs entering it, assembles mines, and sends them to the supply node (the destination node). Although dummy modes represent build rate to maintain consistency in the structure of the mine network, they are not included in the mode network since build-rate arcs can be used every period and there is no limit on availability over all time periods. The MOMAG node in the time-expanded mine network can have only one arc leave it per time period and this single arc must arrive at the supply node to which it is connected. Figure 4 illustrates a MOMAG-supply node split into two nodes: m_2 and s_2.

The following terms are defined to avoid misinterpretation of the problem formulation since they are not part of standard network terminology:

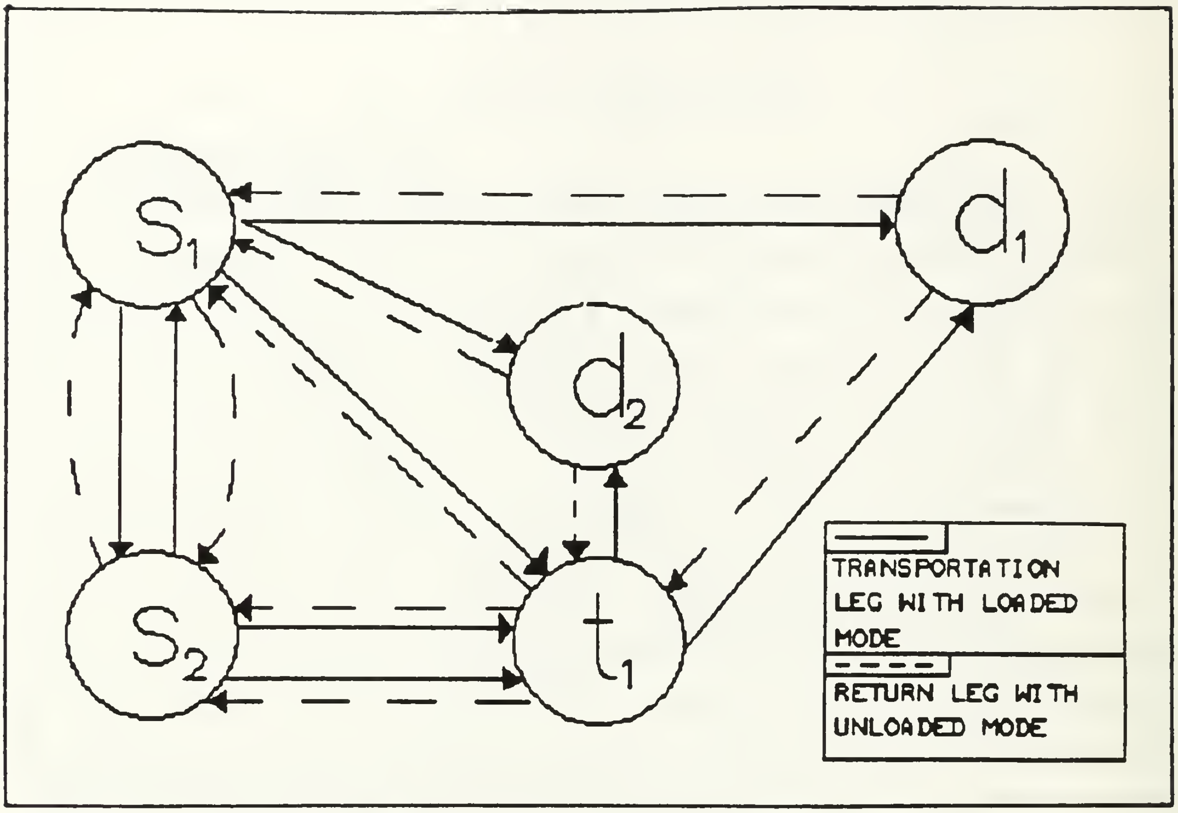
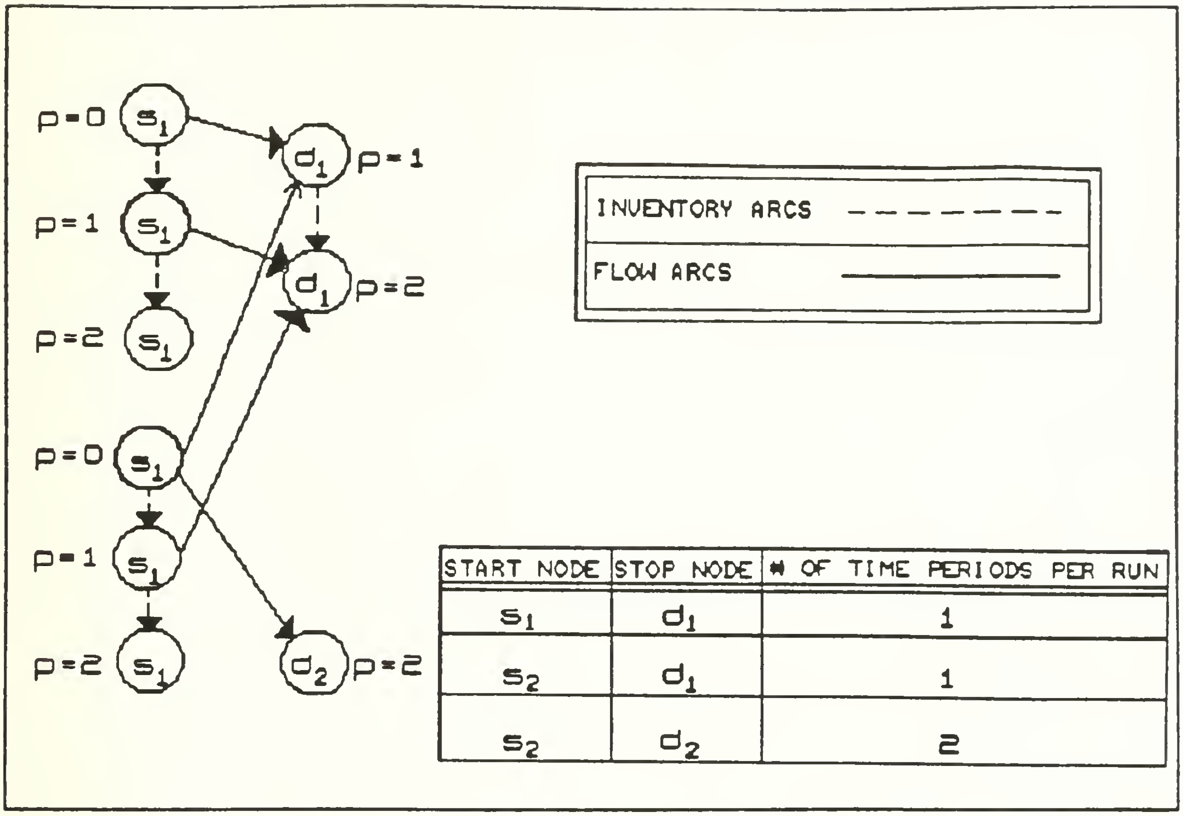

S1
d1
d2
S2
t1
TRANSPORTATION LEG WITH LOADED MODE
RETURN LEG WITH UNLOADED MODE

Figure 2. Example of a Mode Network

- Mode group - a set of modes based at the same node that are grouped by their similarity or by their command structure, such as seven P-3 Orion aircraft based at Naval Air Station (NAS) Adak, Alaska, or a squadron of various aircraft capable of carrying mines deployed aboard an aircraft carrier.
- Mode unit - an element of the mode group, such as one P-3 Orion aircraft or a squadron (if the squadron is composed of different aircraft).
- Run - a round trip from a node to a destination via a mode group.
- Mobile node - a node which is not fixed in position (such as aircraft carriers, surface ships and submarines).
- On-station point - the position from which a naval vessel conducts operations.

B. ASSUMPTIONS

Several assumptions were made in formulating the model to find the optimal mine transshipment schedule. Most assumptions reduce the scope of the problem to decrease the size of the network. The model assumptions are:



START NODE	STOP NODE	# OF TIME PERIODS PER RUN
s_1	d_1	1
s_2	d_1	1
s_2	d_2	2

Figure 3. Example of a Time-Expanded Mine Network

- The model is single-commodity, i.e., only one mine type can be shipped.
- The mine network or the mode network need not be connected. For example, Atlantic-based mines will usually be deployed to mine fields in the Atlantic, Pacific-based mines will usually be laid in mine fields in the Pacific, and there need not be arcs (i.e., routes) connecting the mine and mode networks for the Atlantic fleet to the networks of the Pacific fleet.
- A mode group can have only one destination per run and cannot be split to start more than one run at different times.
- Multiple runs can be made by a mode group to different destinations.
- Two levels of mine readiness are assumed for mines stored at the MOMAG-supply nodes: disassembled and assembled.
- Mobile nodes that cannot deploy mines (such as aircraft carriers and surface ships) are treated as immobile nodes for the duration of the problem. The air modes stationed on the nodes deliver the mines.
- Mobile nodes that cannot deploy mines may transit to an on-station point at the beginning of the problem and to a new destination at the end of the problem. These transit distances must be input by the user. However, no additional transits

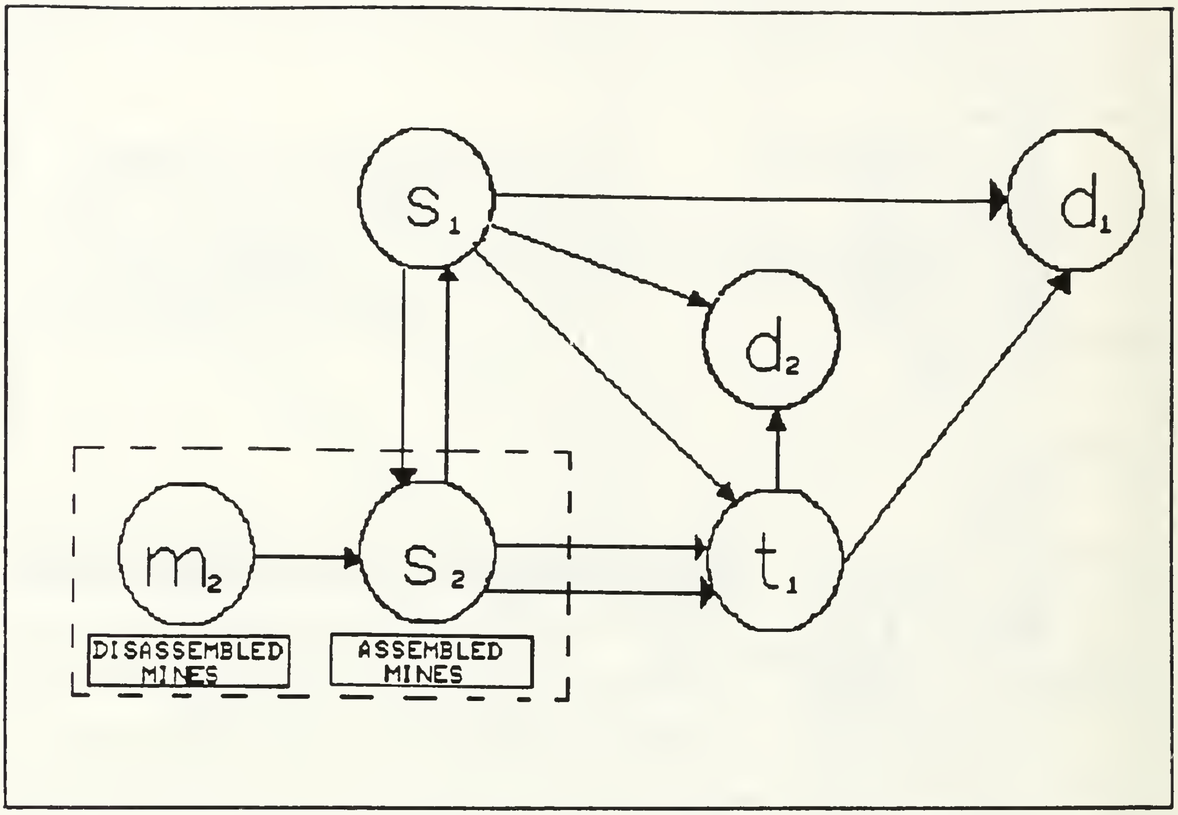


Figure 4. Example of a MOMAG-Supply Node Split in a Mine Network

may be made to reposition a mobile node at other on-station points. A mobile node can only lay mine fields at demand nodes accessible from the on-station point.

- Mobile nodes that can deploy mines (such as submarines) are also treated as immobile nodes with dummy modes attached to them that function as delivery modes. The dummy modes maintain consistency in the structure of the model.
- Mobile nodes cannot be resupplied.
- Several mine fields located in the same area may be grouped into one demand node.
- All modes must return to their origin node before the problem ends.
- A mode that can either transport or deploy mines will always be classified as a delivery mode to eliminate unnecessary arcs.

C. FORMULATION

1. Indices

The first four of the five following indices are used in the formulations of both models, while the last index applies only to the second proposed model:

- n - an element of the set of all nodes in the original network, $N \equiv \{1, 2, \ldots, n, \ldots, n_N\}$, where n_N is the number of nodes.

- μ - an element of the set of all modes in the original network $\mathbf{M} \equiv \{1, 2, \dots, \mu, \dots, n_{\mathrm{M}}\}$, where n_{M} is the number of modes.
- p - an element of the set of all time periods used for the time-expansion, $\mathbf{P} \equiv \{0, 1, 2, \dots, p, \dots, n_{\mathrm{P}}\}$, where n_{P} is the number of time periods plus one.
- i - an element of the set of integers used for iterative loops, $\mathbf{I} \equiv \{1, 2, \dots, i, \dots, n_{\mathrm{I}}\}$, where $n_{\mathrm{I}} = n_{\mathrm{N}}^2$.
- g - an element of the set of incompatible arc groups, $\mathbf{G} \equiv \{1, 2, \dots, g, \dots, n_{\mathrm{G}}\}$, where n_{G} is the highest total number of groups, where $n_{\mathrm{G}} = n_{\mathrm{P}} - 1$.

The elements in the set **P** actually represent the start and end points of time periods considered by the model. For example, the first period starts at p equals 0 and ends at p equals 1. The extra period starting at $n_{\mathrm{P}} - 1$ and ending at n_{P} is needed to calculate constants for mine inventory after the last period of the problem. The procedure for determining incompatible arc groups is described in the section under "Generation of Incompatible Arc Groups."

The node and mode indices can be categorized by several subset indices representing the function, dimension or arc position of the node or mode. For example, MOMAG nodes, which are always origin nodes, can only be positioned on land and transshipment modes travel on land or by air. This information is critical in constructing a network that has realistic arcs and meets the assumptions of the model. The following subindices are subsets of the node set **N**:

- m - an element of the set of all MOMAG nodes **MN** , where $m \in \mathbf{MN} \subset \mathbf{N}$.
- s - an element of the set of all supply nodes **SN** , where $s \in \mathbf{SN} \subset \mathbf{N}$.
- t - an element of the set of all transshipment nodes **TN** , where $t \in \mathbf{TN} \subset \mathbf{N}$.
- d - an element of the set of all demand nodes **DN** , where $d \in \mathbf{DN} \subset \mathbf{N}$.
- l - an element of the set of all nodes positioned on land **LN**, where $l \in \mathbf{LN} \subseteq (\mathbf{MN} \cup \mathbf{SN} \cup \mathbf{TN})$.
- c - an element of the set of all nodes located on or under the sea **CN**, where $c \in \mathbf{CN} \subseteq (\mathbf{SN} \cup \mathbf{DN})$.
- i - an element of the set of all origin nodes **IN** for arcs in the original network, where $i \in \mathbf{IN} \equiv (\mathbf{MN} \cup \mathbf{SN} \cup \mathbf{TN})$.
- j - an element of the set of all destination nodes **JN** for arcs in the original network, where $j \in \mathbf{JN} \equiv (\mathbf{SN} \cup \mathbf{TN} \cup \mathbf{DN})$.

Figure 5 indicates relationships between the function subsets (**MN**, **SN**, **TN**, and **DN**) and the dimension subsets (**LN** and **CN**). Figure 6 depicts the relationships between the function subsets and the arc position subsets (**IN** and **JN**). Notice that the unions of the four mutually-exclusive function subsets and of the dimension subset are identical to the node set N . Finally, the union of the arc position subsets also equals N but their intersection is not empty: $(\mathbf{IN} \bigcap \mathbf{JN}) \equiv (\mathbf{SN} \bigcup \mathbf{TN})$. The only node subset that the user must input in addition to the node set N is the sea node subset **CN**. SUMIT assumes all other nodes are land nodes. The proposed model determines the membership of the function subsets from the input data.

The final group of subindices are the following subsets of the mode set **M**:

- β - an element of the set of all dummy build modes **BM** that transfer mines assembled at MOMAG nodes to adjacent supply nodes, where $\beta \in \mathbf{BM} \subset \mathbf{M}$.
- τ - an element of the set of all transportation modes **TM** that can only transfer mines to transshipment or land supply nodes, where $\tau \in \mathbf{TM} \subset \mathbf{M}$.
- δ - an element of the set of all delivery modes **DM** that can deploy mines into mine fields, where $\delta \in \mathbf{DM} \subseteq \mathbf{M}$.
- λ - an element of the set of all modes **LM** that travel on land, where $\lambda \in \mathbf{LM} \subseteq (\mathbf{BM} \bigcup \mathbf{TM})$.
- γ - an element of the set of all dummy modes **CM** for sea nodes that can deploy mines into mine fields, where $\gamma \in \mathbf{CM} \subseteq \mathbf{DM}$.
- α - an element of the set of all aircraft modes **AM** that transport or deliver mines, where $\alpha \in \mathbf{AM} \subseteq (\mathbf{TM} \bigcup \mathbf{DM})$.

The same dummy build mode can be used for all MOMAG nodes if they have the same build rate (the treatment of parallel assembly lines is mentioned in the next section). Figure 7 indicates the relationship between the function and dimension subsets of **M**. The function subsets (**BM**, **TM**, and **DM**) are mutually-exclusive sets whose union forms the mode set **M**. The union of the mutually-exclusive dimension sets (**LM**, **CM**, and **AM**) also is identical to the mode set **M**. In addition to listing the mode set **M**, the user must input all function subsets and must indicate both sea and land subsets. The proposed model assumes that all remaining modes are air modes.

2. Data

The following list of parameters is a brief description of data contained in both model formulations. All data used in both models is fully described in the section entitled "Detailed Data Description for Both Models." The first parameter is not actually contained in the computer implementation of the models but is created to simplify the presentation of the equations located in the sections of this chapter under "Model A" and "Model B." The value of first parameter, $OBJ_{ijp\mu}$ is not directly input by the user and is calculated by SUMIT from input data.

- $OBJ_{ijp\mu}$- the value contributed to the objective function by the mode flow for mode μ leaving origin node i arriving at destination node j at time period p.

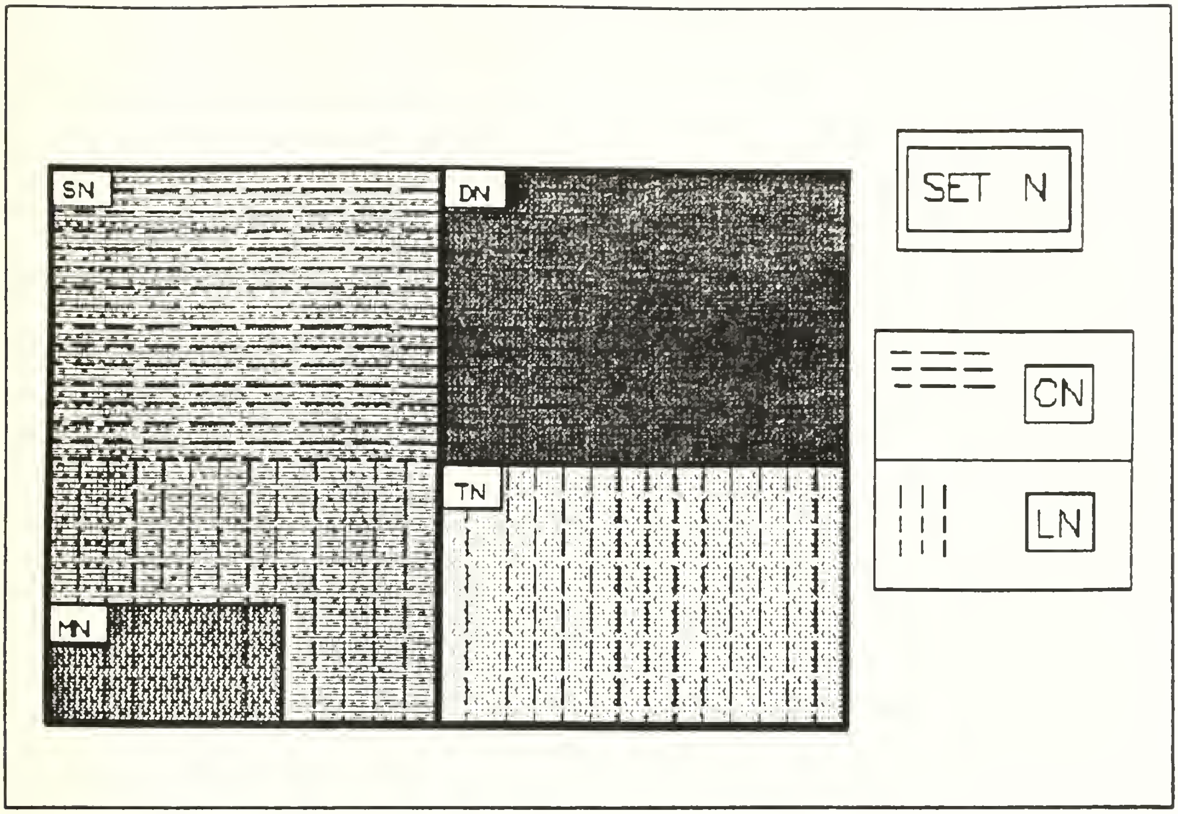


Figure 5. Relationship Between Node Function and Dimension Subsets

- $PAR_{ij\mu}$ - the number of time periods with length TPER required to load mode μ based at origin node i, travel to destination node j and unload (or deliver) mines.
- $PRT_{ij\mu}$ - the number of time periods with length TPER required to load mode μ based at origin node i, travel to destination node j , unload (or deliver) mines, return, and make ground preparations for turnaround.
- AMT_n - the supply or demand of mines at node n.
- $XUP_{ij\mu}$ - the upper bound on mine flow variables from origin node i to destination node j via mode μ.
- ZUP_n - the upper bound on mines in inventory at node n.
- $MXT_{i\mu}$ - the maximum total time (h) that mode group μ can be absent from its origin node i in making all of its runs.
- $MXR_{i\mu}$ - the maximum number of runs that mode group μ based at its origin node i is allowed to take.
- RHX_{np} - the value of the right-hand side of the mine flow balance equation for mines leaving node n at the beginning of time period p, where p is either the time period before inventory can start changing or the time period after inventory can stop changing.

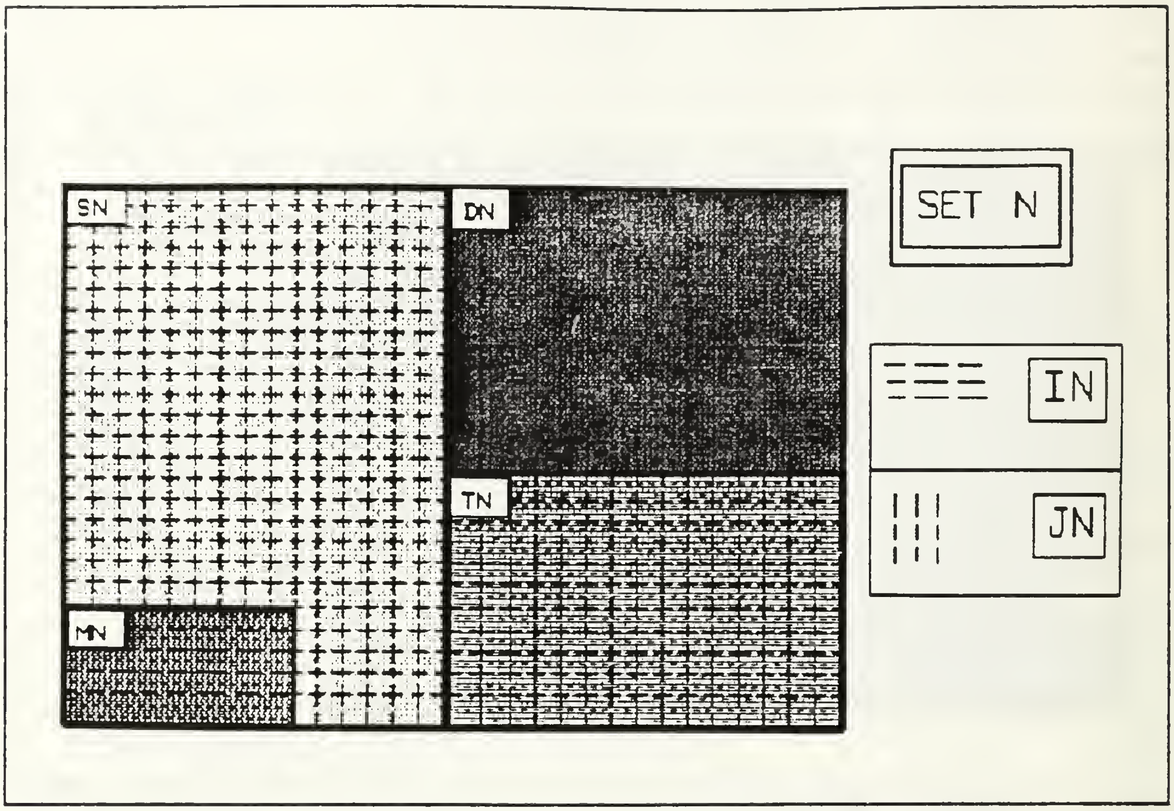


Figure 6. Relationship Between Node Function and Arc Position Subsets

- $RHY_{ip\mu}$ - the value of the right-hand side of the mode flow balance equation for mode μ based at origin node n at the beginning of time period p .

Of the preceding parameters, only AMT_n, $MXT_{i\mu}$, and $MXR_{i\mu}$ are input by the user. The remaining parameters are derived from the input data.

3. Decision Variables

Five decision variables are included in the first proposed model, while only the first four decision variables are included in the second proposed model:

- v - the objective value of the model, which is a measure of effectiveness that involves node and mode priorities and return time for arcs selected by the model.
- $x_{ijp\mu}$ - the number of mines sent from origin node i via mode μ and arriving at destination node j at the end of time period p, where $x_{ijp\mu} \geq 0$.
- $y_{ijp\mu}$ - a binary variable that equals 1 if $x_{ijp\mu} > 0$ and equals 0 otherwise.
- $\hat{x}_{np}$ - the number of mines remaining in inventory at node n at the end of time period p .
- $\hat{y}_{ip\mu}$ - a binary variable that equals 1 if all of mode units in mode group μ remained at origin node i during time period p and 0 otherwise.

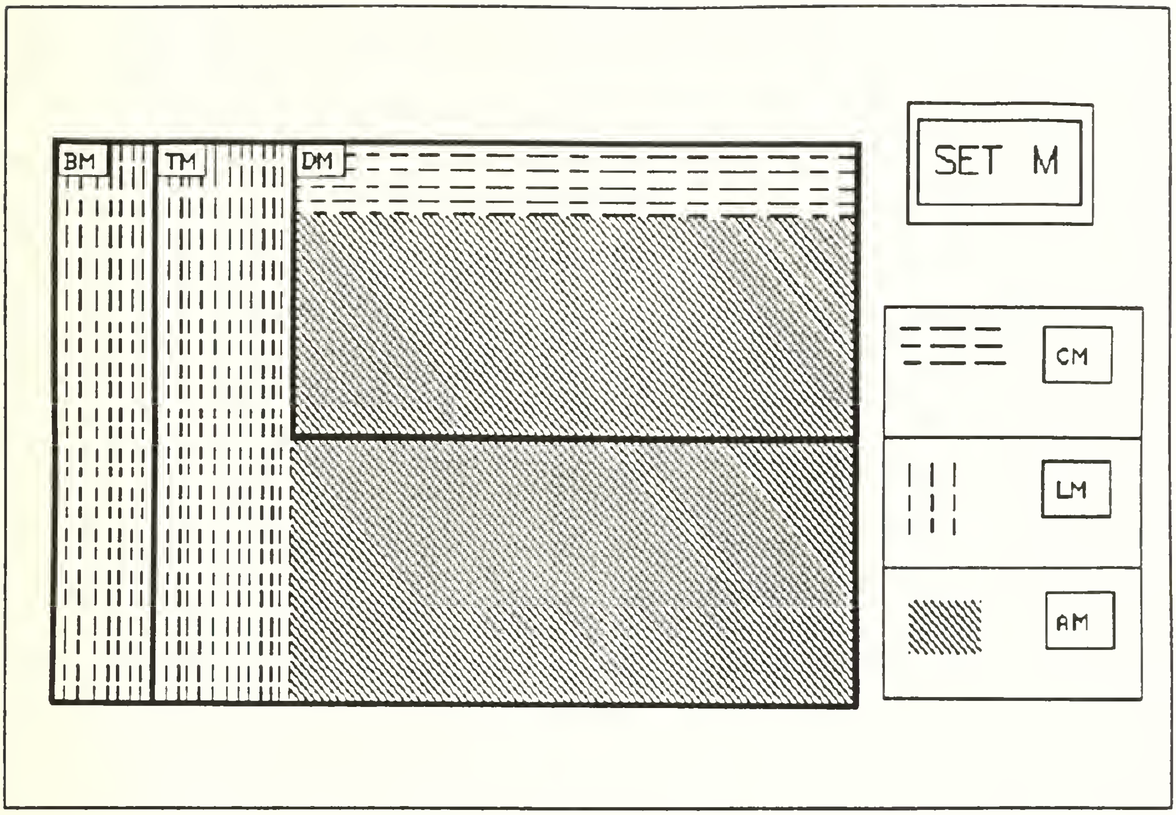


Figure 7. Relationship Between Mode Function and Dimension Subsets

Mine flow and inventory is represented by positive, continuous variables. Since, for fixed values of $y_{ijp\mu}$, the resulting problem is a single-commodity network flow problem with integer supplies, demands, and bounds on flows, these continuous variables are guaranteed to be integer [Ref. 9]. The value of $y_{ijp\mu}$ indicates whether or not mode μ is used for the arc from i to j arriving at period p and does not reflect the number of mode units within mode group μ that are actually needed to carry the mines. Likewise, $\hat{y}_{ip\mu}$ indicates whether or not the entire mode group remains at node i during time period p. In some cases, the model solution may imply that some, but not all mode units within a mode group, will be empty as they transit. By the assumptions of the model, these empty mode units cannot be diverted to meet other mine field demands. However, in reality, if the scenario warrants it (i.e., squadron integrity is not required for defense purposes), the empty mode units can remain at the origin node or be diverted to accomplish missions, not related to the problem.

4. Model A

Model A, the first model proposed in this thesis, consists of the objective equation, flow balance equations, linking equations, other constraints and bounds on the variables, which are explained in the section after the formulation. **XTE** and **YTE** are two sets that represent the variables associated with the arcs that exist in the time-expanded networks. **XTE** is the set of all variables in the mine network, while **YTE** is the set of all variables representing flow from origin nodes to destination nodes in the mode network. Rules that explain membership in these two sets are explained in the section under "Flow Existence Rules." The time periods p^1 , p^2, and p^3 are defined as follows for applicable equations in the model formulation:

- $p^1 = p + 1.$
- $p^2 = p + PAR_{nj\mu}.$
- $p^3 = p - PRT_{in\mu} + PAR_{in\mu}.$

Minimize

$$v = \sum_{i=1}^{n_N} \sum_{j=1}^{n_N} \sum_{p=0}^{n_P} \sum_{\mu \ni y_{ijp\mu} \in \mathrm{YTE}} OBJ_{ijp\mu} y_{ijp\mu} \tag{1}$$

Subject to

Inflow from all nodes

outflow from

$$\hat{x}_{np^1} - \hat{x}_{np} + \sum_{j=1}^{n_N} \sum_{\mu \ni x_{njp^2\mu} \in \mathrm{XTE}} x_{njp^2\mu} - \sum_{i=1}^{n_N} \sum_{\mu \ni x_{inp\mu} \in \mathrm{XTE}} x_{inp\mu} = RHX_{np} \quad \forall n \in \overline{\mathbf{DN}}, p \tag{2}$$

Starting

$$\hat{x}_{dp^1} - \hat{x}_{dp} - \sum_{i=1}^{n_N} \sum_{\mu \ni x_{idp\mu} \in \mathrm{XTE}} x_{idp\mu} = RHX_{dp} \quad \forall d, p \tag{3}$$

$$\hat{y}_{np^1} - \hat{y}_{np} + \sum_{j \ni y_{njp^2\mu} \in \mathrm{YTE}} y_{njp^2\mu} - \sum_{i \ni y_{inp^3\mu} \in \mathrm{YTE}} y_{inp^3\mu} = RHY_{np\mu} \quad \forall n, p, \mu \tag{4}$$

$$XUP_{ijp\mu}\, y_{ijp\mu} - x_{ijp\mu} \geq 0 \quad \forall i, j, p, \mu \ni x_{ijp\mu} \in \mathbf{XTE},\ y_{ijp\mu} \in \mathbf{YTE} \tag{5}$$

$$\sum_{j=1}^{n_N} \sum_{p \ni y_{ijp\mu} \in \mathrm{YTE}} PRT_{ij\mu}\, y_{ijp\mu} \leq MXT_{i\mu} \quad \forall i, \mu \tag{6}$$

$$\sum_{j=1}^{n_N} \sum_{p \ni y_{ijp\mu} \in \mathrm{YTE}} y_{ijp\mu} \leq MXR_{i\mu} \quad \forall i, \mu \tag{7}$$

$$0 \leq x_{ijp\mu} \leq XUP_{ij\mu} \quad \forall i, j, p, \mu \tag{8}$$

$$y_{ijp\mu} \in \{0, 1\} \quad \forall i, j, p, \mu \tag{9}$$

$$0 \leq \hat{x}_{np} \leq ZUP_n \quad \forall n, p \tag{10}$$

$$\hat{y}_{ip\mu} \in \{0, 1\} \quad \forall i, p, \mu \tag{11}$$

5. Comments about Model A

The following list of comments explains the purpose of the equations given in the preceding model formulation:

- Equation 1 is the equation for the objective value.
- Equations 2 and 3 are mine flow balance equations for every node and time period in which mine flow is possible. Equation 2 is for nondemand nodes and Equation 3 for demand nodes.
- Equation 4 is the mode flow balance equation for every origin node, mode and time period in which mode flow is possible.
- Equation 5 links the mine and mode networks by forcing the mode flow variable $y_{ijp\mu}$ to 1 if the mine flow variable $x_{ijp\mu}$ is positive.
- Equation 6 is optional and ensures that the total time that a given mode is away from a given base is less than $MXT_{i\mu}$.
- Equation 7 is optional and ensures that the total number of runs for a given mode from a given base is less than $MXR_{i\mu}$.
- Equations 8 - 11 ensure that all variables are bounded.

In the computer code for the model, the equal signs are relaxed (i.e., are changed to inequalities) in Equations 1 - 4 to make the model easier to solve. The equal sign in Equation 1 is changed to greater than or equal to ($\geq$), while the equal signs in Equations 2 - 4 are changed to less than or equal to ($\leq$).

6. Model B

Model B is an alternate model proposed for SUMIT. It is based on the premise that, by only allowing one mode arc in a group of incompatible mode arcs to be selected by the model, the model can monitor the movement of modes correctly and prevent a mode group from taking-off for its next run before the mode group has returned from its previous run, without using binary variables for mode inventory. The alternate model has one additional index (g) and more internal data, eliminates the need for the mode inventory variable $\hat{y}_{np\mu}$, replaces Equation 4 with a single equation, and deletes Equation 11. The alternate model makes the same assumptions as the original model. The network generation rules are also the same and the mine and mode flow existence rules are modified. The set, $\mathbf{IG}_g$, represents the incompatible arcs derived for group g. The generation of the set $\mathbf{IG}_g$ is described more fully in this chapter under "Generation of Incompatible Arc Groups."

Minimize

$$v = \sum_{i=1}^{n_N} \sum_{j=1}^{n_N} \sum_{p=0}^{n_P} \sum_{\mu \ni y_{ijp\mu} \in \mathrm{YTE}} OBJ_{ijp\mu}\, y_{ijp\mu} \qquad (1)$$

Subject to

$$\hat{x}_{np^1} - \hat{x}_{np} + \sum_{j=1}^{n_N} \sum_{\mu \ni x_{njp^2\mu} \in \mathrm{XTE}} x_{njp^2\mu} - \sum_{i=1}^{n_N} \sum_{\mu \ni x_{inp\mu} \in \mathrm{XTE}} x_{inp\mu} = RHX_{np} \quad \forall n \in \overline{\mathbf{DN}}, p \qquad (2)$$

$$\hat{x}_{dp^1} - \hat{x}_{dp} - \sum_{i=1}^{n_N} \sum_{\mu \ni x_{idp\mu} \in \mathrm{XTE}} x_{idp\mu} = RHX_{dp} \quad \forall d, p \qquad (3)$$

$$\sum_{j=1}^{n_N} \sum_{p \ni y_{ijp\mu} \in \mathrm{YTE} \cap \mathrm{IG}_g} y_{ijp\mu} \leq 1 \quad \forall i, \mu, g \qquad (12)$$

$$XUP_{ijp\mu}\, y_{ijp\mu} - x_{ijp\mu} \geq 0 \quad \forall i, j, p, \mu \ni x_{ijp\mu} \in \mathbf{XTE},\ y_{ijp\mu} \in \mathbf{YTE} \qquad (5)$$

$$\sum_{j=1}^{n_N} \sum_{p \ni y_{ijp\mu} \in \mathrm{YTE}} PRT_{ij\mu}\, y_{ijp\mu} \leq MXT_{i\mu} \quad \forall i, \mu \tag{6}$$

$$\sum_{j=1}^{n_N} \sum_{p \ni y_{ijp\mu} \in \mathrm{YTE}} y_{ijp\mu} \leq MXR_{i\mu} \quad \forall i, \mu \tag{7}$$

$$0 \leq x_{ijp\mu} \leq XUP_{ij\mu} \quad \forall i, j, p, \mu \tag{8}$$

$$y_{ijp\mu} \in \{0, 1\} \quad \forall i, j, p, \mu \tag{9}$$

$$0 \leq \hat{x}_{np} \leq ZUP_n \quad \forall n, p \tag{10}$$

7. Comments about Model B

All comments from Model A also apply to Model B, except for those that pertain to Equations 4 and 11. The purpose of Equation 12, which replaces Equation 4 in Model A, is to ensure that only one arc in a group of incompatible arcs is selected.

8. Detailed Data Description for Both Models

The data discussed in the next five paragraphs is input by the user and must be manipulated by both models to generate the networks, calculate the data describing the networks, time-expand the nodes to form the time-expanded networks, and develop data for the objective and constraint equations. The input data in the following list is scalar. The first three scalars are required input and the last three are optional (default values will be assumed if no input is given):

- $TEND$ - the time (in hours, h) by which all modes transport or deliver mines, meeting all demand, and return to their origin nodes, where $TEND > 0$.
- $TPER$ - the length of the time period (h) used in the time expansion, where $0 < TPER \leq TEND$.
- $RELT$ - the value of the zero tolerance for comparing real data, where $0 < RELT \leq 0.001$. I.e, if $|x - y| < RELT$, then $x = y$ is assumed.
- $MNLA$ - the minimum distance (in nautical miles, nm) that air modes can transport mines to prevent air modes from shipping mines over short routes intended for land modes, where $MNLA \geq 0$. The default is 0.
- $MXSS$ - the maximum distance (nm) allowed for supply to supply transfers, where $MXSS > 0$. The default is the maximum distance between nodes.

- *DISC* - a binary parameter in which 1 means to run the model without checking for disconnected networks and 0 means to check for disconnected networks and to run the model for the first connected network found. The default is 0.

The model converts *TEND* into $\lceil TEND/TPER \rceil$ time periods since it time-expands the network over time periods of length *TPER*. If *TPER* is relatively small, the round-off error is less when converting times to time periods. However, a smaller time period length also yields more time periods for the problem, which expands the size of the model. Since a large model is harder to solve, a balance must be struck between *TPER* and the number of periods that will be formed. The user must also make *TEND* large enough to result in a feasible solution while making it small enough to cut down the size of the time-expansion.

The data presented in the following list pertains to the node index *n* or any of its subindices. For all data pertaining to priorities in this thesis, *a lower value implies a higher priority*, e.g., a demand node with a priority of 1 is more important than a node with priority 2. The purpose of the third parameter MXD_n is to allow the user to group mine fields that are close together into one node and to account for transit time needed to travel between mine fields. Since a mode group can only have one destination node per run, MXD_d permits the mode group to travel to all mine fields within a demand node if it has the capacity to carry enough mines. The first parameter is required as input and the last five are optional:

- AMT_n - the supply or demand at node *n* (mines), where, by convention, assembled supply and MOMAG disassembled supply amounts are positive and demand amounts are negative. The default is 0.
- PRN_n - the priority of node *n*, where $PRN_n > 0$. The default is the maximum node priority (or 1 if none are input).
- MXD_d - the maximum distance (nm) between mine fields within a demand node *d*, where $MXD_d \geq 0$. The default is 0.
- BGD_c - the distance (nm) that must be transited by the sea node *c* to reach its on-station point before launching aircraft to lay mines, where $BGD_c \geq 0$. The default value is 0.
- EDD_c - the distance (nm) that must be transited by the sea node *c* to reach a new destination by the last period of the problem after it completes its last run, where $EDD_c \geq 0$. The default value is 0.
- SPX_c - the transit speed (nm/h) of the sea node, where $SPX_c > 0$. If $BGD_c > 0$ or $EDD_c > 0$, SPX_c is no longer optional.

The optional parameters BGD_c, EDD_c, and SPX_c are not intended to be used for submarines and HVMLs since these modes transit directly to the mine field.

The following list of input parameters describes mode characteristics indexed by mode μ or any of its subindices. The first seven parameters are required and the last one is optional:

- CAP_μ - the capacity of one mode unit of mode μ (mines), where $CAP_\mu > 0$. CAP_μ is required for all modes except dummy build modes, whose capacities are computed by the model.
- MXL_μ - the maximum distance (nm) that mode μ can travel from its origin node and be able to return, where $MXL_\mu > 0$. MXL_μ is required for all modes except the dummy build modes.
- SPD_μ - the average transit speed (nm/h) of mode μ , where $SPD_\mu > 0$. SPD_μ is required for all modes, where the speed of a dummy build mode β is its build rate.
- TLD_μ - the average amount of time (h) needed to load a mode group and take-off, where $TLD_\mu \geq 0$. The default is 0.
- TUL_μ - the average amount of time (h) needed to unload a mode group for transportation modes or deploy the mines for delivery mode groups, where $TUL_\mu \geq 0$. The default is 0.
- TGD_μ - the average amount of time (h) needed to spend on the ground after a run before reloading and taking off, where $TGD_\mu \geq 0$. The default is 0.
- PRM_μ - the priority of mode μ, where $PRM_\mu > 0$. The default is the maximum mode priority (or 1 if none are input).

The parameter CAP_μ can be used in two different ways. If a mode group is composed of the same type of mode unit, the capacity of a mode unit can be input as CAP_μ. But, if the mode group is composed of variety of modes, CAP_μ should be the total number of mines that the mode group can carry. The user must take this distinction into account when inputting the number of modes stationed at node. For example, if the mode group at a give node is composed of seven P3 Orion aircraft and the user inputs CAP_μ as the capacity of a mode unit, the number of modes at that node should be seven. However, if the mode group based at a given node is a tactical air squadron and the user inputs CAP_μ as the total capacity, the number of modes at that node should be one. For dummy sea modes based at mobile sea nodes that can deploy mines, CAP_γ should equal the supply of the node in which the dummy sea mode is stationed. The three time-related parameters, TLD_β, TUL_β, and TGD_β , are assumed to equal 0 for dummy build modes.

The next list of parameters involves the mode groups based at their origin nodes. The first parameter is required and the last four parameters are optional:

- $NUM_{i\mu}$ - the number of mode units μ in the mode group based at origin node i, where $NUM_{i\mu} \geq 0$. $NUM_{i\mu} \geq 1$ for at least one mode group containing mode units μ at every origin node i. The default is 0.

- $BGT_{i\mu}$ - the earliest time (h) in which mode group μ is available for loading at its origin node i for its first run, where $0 \leq BGT_{i\mu} \leq TEND$. The default is 0.
- $EDT_{i\mu}$ - the latest time (h) by which mode group μ must return to its origin node i after completing its last run, where $0 \leq EDT_{i\mu} \leq TEND$. The default is TEND.
- $MXT_{i\mu}$ - the maximum total time (h) that mode group μ can be absent from its origin node i in making all of its runs, where $0 \leq MXT_{i\mu} \leq TEND$. The default is $TEND$.
- $MXR_{i\mu}$ - the maximum number of runs that mode group μ based at its origin node i is allowed to take, where $0 \leq MXR_{i\mu} \leq [TEND \,/\, TPER]$. The default is $[TEND \,/\, TPER]$.

For dummy build modes, $NUM_{m\beta}$ should equal 2 if the MOMAG node m has dual parallel assembly lines and 1 otherwise. This convention will double the build rate for parallel assembly lines. If the number of parallel assembly lines at MOMAGs are increased in the future, then the user can account for this growth by setting $NUM_{m\beta}$ equal to the number of parallel assembly lines. For dummy sea modes stationed on mobile sea nodes that can deploy mines, $NUM_{c\gamma}$ should equal 1.

The following optional input parameter pertains to arcs in the mine and mode networks. It allows the user to rule out arcs in the networks that will be selected by the model:

- $XMP_{ij\mu}$ - a binary parameter in which 1 implies that no arc from origin node i to destination node j via mode group μ shall be allowed in the network. The default value is 0.

Thus, by including $XMP_{ij\mu}$ if necessary, the user can eliminate arcs that are not allowed because of obscure rules not accounted for in the model.

The final input parameter is required and defines the distance between nodes:

- DIS_{ij} - the distance (nm) between origin node i and destination node j. The default value is 0.

Distances may be given in a table containing all distances between points. Distances may also be listed separately for arcs that will probably be used in the network. If the distance from node i to node j is input, then the model assumes that it also equals the distance from node j to node i. The former method of input ensures that the model will consider all possible node combinations for the networks, while the latter method is less tedious to enter if the user thinks only a few arcs are needed.

Two important conventions must be followed to ensure the successful generation of the correct mine and mode networks. Entering "negative" distances allows the user to describe two properties of the networks without creating additional input parameters.

First, all distances between nodes that are on the same land mass must be less than -0.5 to indicate arcs that have potential land routes: $DIS_{ll'} < -0.5$, where $l \neq l'$. Second, to identify a MOMAG node and its adjacent supply node, the distance between a MOMAG node and its adjacent supply node must be equal to -0.5: $DIS_{ms} = -0.5$. This convention ensures that the MOMAG node is connected only to its adjacent supply node.

The next list of parameters is computed by the model, given the input data to describe the original and the time-expanded networks for mode flow and mine flow. These parameters assume that the network has already been generated. (Generation rules will be given in the next section of this chapter under "Network Generation Rules"). The model takes a conservative approach to rounding time periods up or down. For example, when converting time data to time periods, the model rounds up begin times input by the user ($BGT_{i\mu}$) but rounds down end times ($EDT_{i\mu}$). Rounding data in this way ensures that the time periods fall within time bounds input by the user. Likewise, calculations of arrival time periods for one-way trips and return time periods for round trips are rounded up to be on the safe side.

- $NRLT$ - the value of the negative zero tolerance allowed when comparing real data, where $NRLT = -RELT$.
- $SUMA$ - the number of arcs in the mine network.
- $ARC_{ij\mu}$ - the set that indicates whether or not the arc from node i to node j via mode μ belongs in the mine and modes networks.
- $ART_{ij\mu}$ - a temporary set that is identical to $ARC_{ij\mu}$.
- DNO_n - a temporary parameter that serves two different purposes in the model: (1) to indicate when node n has already been checked for calculating MXM_n in one loop and for calculating MXS_n in another loop and (2) to indicate that node n is included in the first connected network found.
- NFC_n - the set of nodes that does not form a component with at least one supply node and one demand node.
- $XTE_{ijp\mu}$ - the set that indicates whether or not the mine arc leaving node i and arriving at j at the end of time period p via mode μ, belongs in the time-expanded network.
- $YTE_{ijp\mu}$ - the set that indicates whether or not the mode arc leaving node i and arriving at j at the end of time period p via mode μ, belongs in the time-expanded network.
- MNU_{np} - the set of all mine flow balance equations in which at least one arc leaves node n at the beginning of time period p or one arc arrives at node n at the end of time period p.

- $MDU_{ip\mu}$ - the set of all mode flow balance equations in which at least one arc leaves node i at the beginning of time period p via mode μ or one arc arrives at node i at the end of time period p via mode μ.

- RHX_{np} - the value of the right-hand side of the mine flow balance equation for mines leaving node n at the beginning of time period p, where p is either the time period before inventory can start changing or the time period after inventory can stop changing and

$$RHX_{np} = \begin{cases} \min\{|AMT_n|, ZUP_n\} & \text{if } p \text{ is before.} \\ 0 & \text{if } p \text{ is after.} \end{cases}$$

- $RHY_{ip\mu}$ - the value of the right-hand side of the mode flow balance equation for mode μ based at origin node n at the beginning of time period p , where $RHY_{ip\mu}$ equals 1 if p is the time period before mode μ can start its first run, equals -1 if p is the time period after it can finish its last run, and equals 0 otherwise. Notice that, since mode flow and inventory variables are binary, 1 represents the presence of the supply of mode μ at n when the problem starts and -1 represents the final return of the mode supply at the end of the problem.

- $PAR_{ij\mu}$ - the number of time periods with length TPER required to load mode μ based at origin node i, travel to destination node j and unload (or deliver) mines, where

$$PAR_{ij\mu} = \frac{TLD_\mu + TUL_\mu}{TPER} + \begin{cases} 1 & \text{if } \mu \in \mathbf{BM}. \\ (|DIS_{ij}| + MXD_i) \,/\, (SPD_\mu\ TPER) & \text{otherwise.} \end{cases}$$

- $PRT_{ij\mu}$ - the number of time periods with length TPER required to load mode μ based at origin node i, travel to destination node j , unload (or deliver) mines, return, and make ground preparations for turnaround, where

$$PRT_{ij\mu} = \frac{TLD_\mu + TUL_\mu + TGD_\mu}{TPER} + \begin{cases} 1 & \text{if } \mu \in \mathbf{BM}. \\ (2\ |DIS_{ij}| + MXD_i) \,/\, (SPD_\mu\ TPER) & \text{otherwise.} \end{cases}$$

- IND_n - the indegree of (or the number of arcs directed into) node n.

- OTD_n - the outdegree of (or the number of arcs directed out of) node n.

- $BGP_{ij\mu}$ - the earliest time period in which a run can start at origin node i and arrive at its destination node j via mode μ, where $BGP_{ij\mu} \geq BGT_{i\mu} / TPER$.

- $EDP_{ij\mu}$ - the latest time period in which a run can start at origin node i and arrive at its destination node j via mode μ, where $EDP_{ij\mu} \leq EDT_{i\mu} / TPER$.

- MXM_n - the maximum number of mines that could be sent through node n if all demand on paths containing node n were filled.

- MXS_n - the maximum number of mines that could be sent through node n if all supply on paths containing node n were sent.

- $OBJ_{ijp\mu}$- the value contributed to the objective function by the mode flow for mode μ leaving origin node i arriving at destination node j at time period p, where

$$OBJ_{ijp\mu} = [PRN_i\, PRN_j\, PRM_\mu\, (p + PRT_{ij\mu})]^2 .$$

- ZUP_n - the upper bound (mines) on mine inventory for node n, where
  $$ZUP_n = \min\{MXM_n, MXS_n\}.$$
- $XUP_{ij\mu}$ - the upper bound on mine flow variables from origin node i to destination node j via mode μ, where
  $$XUP_{ij\mu} = \min\{ZUP_i, ZUP_j, CAP_\mu\, NUM_{i\mu}\}.$$

The parameters $BGP_{ij\mu}$ and $EDP_{ij\mu}$ are the keys to eliminating unnecessary arcs. For example, if the origin node i is a transshipment node, the model starts creating outgoing arcs after the earliest period in which mines could have arrived at node i. Likewise, the model does not create arcs that send mines to an origin node in periods after the last possible run could have left the origin node. This requirement cuts down on the number of continuous and binary variables, which increases the speed of model execution.

The parameter $OBJ_{ijp\mu}$ is composed of two parts: the priority parameters and the time parameter. The priority parameters, PRN_i, PRN_j, and PRM_μ, enable the model to consider the relative priorities of nodes and modes during optimization. Because scaling problems may occur if the objective value grows too large, the user should only input *relative priorities* and the maximum priority should be less than five. Since the model is a minimization, the model tends to select mode flow across arcs for which the priority portion of the objective value is smaller (implying that the priority is higher). The second part of the objective value is the time parameter, $p + PRT_{ij\mu}$. The model will again tend to select mode flow for which the time portion is smaller. Thus, mode flow that starts sooner and has a faster round trip time will be considered more optimal. Because it is the product of the priority parameters and time parameters, the objective value of SUMIT incorporates both priority and time into a measure of effectiveness.

Model B generates nine extra internal parameters and deletes one internal parameter ($MDU_{ip\mu}$) from the original model. These additional parameters are needed to delineate the set of incompatible arcs $\mathbf{IG}_g$. Additional parameters generated by Model B are:

- $IBEG$ - a scalar which is incremented by one in the loop that computes $GBG_{ijp\mu}$ and $GED_{ijp\mu}$ for arcs in the incompatible arc groups $\mathbf{IG}_g$, where
  $$IBEG \geq MNB_{i\mu}.$$
- $MRN_{i\mu}$ - the minimum return period $PRT_{ij\mu}$ over all arcs from node i directed to node j via mode μ.

- $MRX_{i\mu}$ - the maximum return period $PRT_{ij\mu}$ over all arcs from node i directed to node j via mode μ.
- $MNA_{i\mu}$ - the minimum arrival period $PAR_{ij\mu}$ over all arcs from node i directed to node j via mode μ for which $PRT_{ij\mu} = MNA_{i\mu}$.
- $MNB_{i\mu}$ - the minimum begin period $BGP_{ij\mu}$ over all arcs from node i directed to node j via mode μ.
- $MXE_{i\mu}$ - the maximum end period $EDP_{ij\mu}$ over all arcs from node i directed to node j via mode μ.
- $LSG_{i\mu}$ - the number of incompatible arc groups leaving i via mode μ, where $EDG'_{ij\mu}$ is $EDG_{ij\mu}$ for which $PAR_{ij\mu} = MNA_{i\mu}$ and $PRT_{ij\mu} = MRA_{i\mu}$ in the equation $$LSG_{i\mu} = \min_{j \in N}\{EDG'_{ij\mu}\} + 2 - MNB_{i\mu} - MRN_{i\mu}.$$
- $GBG_{ijp\mu}$ - the group number g of the first incompatible arc grouping $\mathbf{IG}_g$ in which the arc from i via mode μ to j arriving at time period p appears.
- $GED_{ijp\mu}$ - the group number g of the last incompatible arc grouping $\mathbf{IG}_g$ in which the arc from i via mode μ to j arriving at time period p appears.

9. Network Generation Rules for Both Models

Before discussing the mine and mode flow existence sets **XTE** and **YTE**, rules for generating the original network are given since membership in the mine or mode networks is a criteria for membership in the flow existence sets. Initially, the set of eligible arcs is assumed to be the power set $\mathbf{N} \times \mathbf{N} \times \mathbf{M}$. Arcs that do not meet the rules for network generation are eliminated from the power set. The following list describes the existence requirements for arcs from origin node i to destination node j via mode μ that belong to the original network:

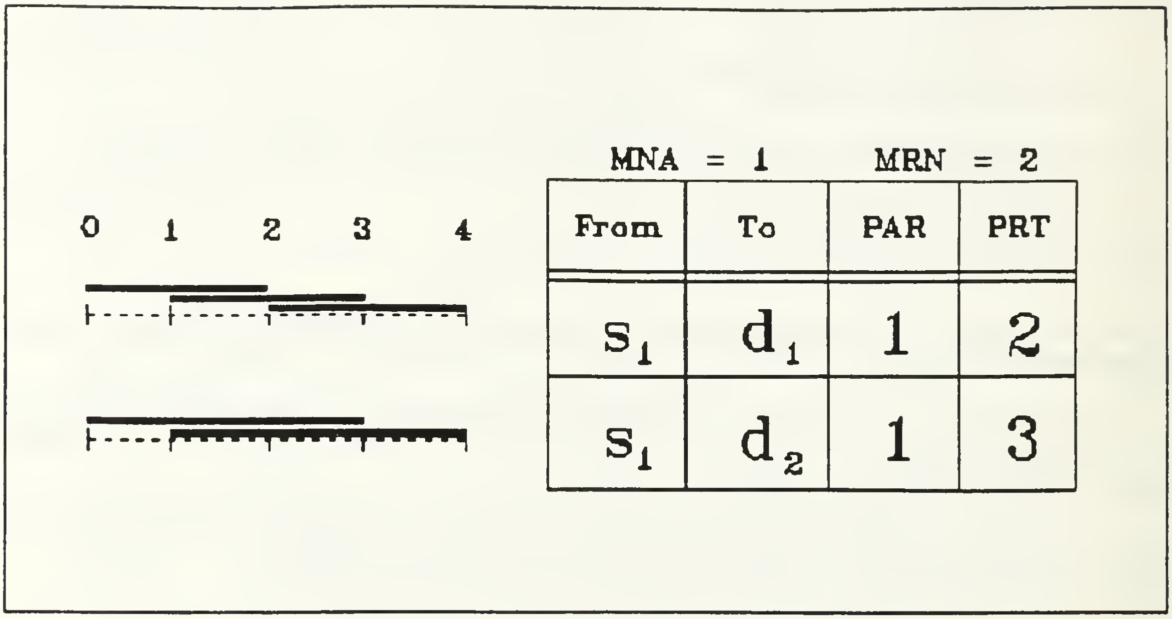
- An arc must leave origin node i and arrive at destination node j via mode μ if $NUM_{i\mu} > 0, i \neq j, DIS_{ij} \neq 0$, and $MXL_{\mu} \leq |DIS_{ij}|$.
- Any arc from i to j via μ is eliminated if $XMP_{ij\mu} = 1$.
- Any arc between two supply nodes on land, $i,j \in \mathbf{SN} \cap \mathbf{LN}$, via a land transportation mode, $\mu \in \mathbf{TM} \cap \mathbf{LM}$, is eliminated if $DIS_{ij} \geq 0$ or $|DIS_{ij}| \geq MXSS$. .
- Any arc between two supply nodes on land, $i,j \in \mathbf{SN} \cap \mathbf{LN}$, via an air transportation mode $\mu \in \mathbf{TM} \cap \mathbf{AM}$ is eliminated if $DIS_{ij} < 0$ and one of the following statements are true: $|DIS_{ij}| \geq MNLA$ or $|DIS_{ij}| \geq MXSS$.
- Any arc between two supply nodes, $i,j \in SN$, via mode μ is eliminated if $i,j \notin \mathbf{LN}$ or $\mu \notin \mathbf{TM}$.
- Any arc is eliminated between two nodes i,j via land transportation mode $\mu \in \mathbf{TM} \cap \mathbf{LM}$ if $DIS_{ij} \geq 0$.
- Any arc is eliminated between two land nodes $i,j \in \mathbf{LN}$ via air transportation mode $\mu \in \mathbf{TM} \cap \mathbf{AM}$ if $DIS_{ij} < 0$ and $|DIS_{ij}| \geq MNLA$.

- Any arc from sea node c is eliminated if it does not arrive at demand node d.
- Any arc directed to either supply node s or transshipment node t is eliminated if the mode is a delivery mode δ.
- Any arc arriving at a demand node d via a nondelivery mode is eliminated.
- Any arc leaving a MOMAG node m and arriving at any node j for which $DIS_{mj} \neq -0.5$ is eliminated.
- Any arc entering a MOMAG node m is eliminated.
- Any arc leaving a transshipment node t and arriving at a supply node s is eliminated.
- Any arc from i to j via μ is eliminated if its round trip time $TPER \times PRT_{ij\mu}$ is greater than the total time allowed $MXT_{i\mu}$ or the time difference between $EDT_{i\mu}$ and $BGT_{i\mu}$.
- Eliminate all arcs directed from node i if node i has no mines in inventory at the beginning of the problem and no arcs are directed into node i.
- Any arc from i to j via μ is eliminated if $BGP_{ij\mu} \geq EDP_{ij\mu}$.
- Any arc whose origin node i and destination node j is not contained in the first connected component found.

10. Generation of Incompatible Arc Groups

The procedure for determining incompatible arc groups for Model B sets up incompatible arc groups around an arc that has a return period equal to $MRN_{i\mu}$ and has an arrival period equal to $MNA_{i\mu}$. Because such an arc has the minimum return period and the minimum arrival period for mode μ leaving node i, the first run associated with this arc has the earliest arrival time period. The number of potential runs for this arc is also greater than (or equal to) the number of potential runs for other arcs that do not meet this criteria because arcs with greater return periods are forced to make fewer runs within the same amount of time. Figure 8 illustrates this point by depicting the relationship between arcs leaving the same node via the same mode arriving at different destinations with different arrival and return time periods. Focusing on this arc, then forms the greatest possible number of incompatible arc groups for mode μ leaving i since it has the most runs and covers all possible time periods since its first run also has the earliest arrival time period.

In the first step of the procedure, incompatible arcs groups are generated for arcs from i via mode μ to j arriving at the end of time period p *with return periods of* $MRN_{i\mu}$ *and with arrival periods of* $MRA_{i\mu}$. Incompatible arcs for an arc from node i arriving at node j by the end of time period p are those that leave i before the mode group can return to i_N. Then, any arc that leaves node i after period $p - MNA_{i\mu}$ and before



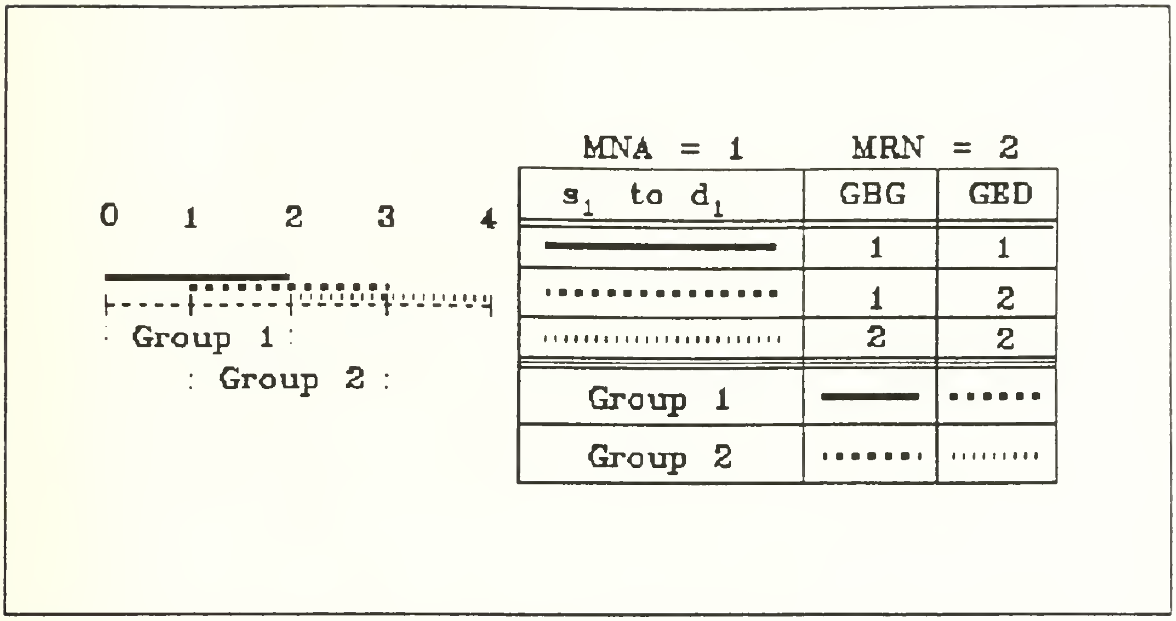
MNA = 1 MRN = 2

From	To	PAR	PRT
s_1	d_1	1	2
s_1	d_2	1	3

Figure 8. Relationship between Arcs from the Same Node to Different Destinations

period $p + MRN_{iu} - MNA_{iu}$, is not compatible. Adding MNA_{iu} to both sides implies that incompatible arcs arrive at node j after period p and before period $p + MRN_{iu}$. The procedure starts at $IBEG = MNB_{iu}$ for the first group, iterates through a loop that calculates GBG_{ijpu} and GED_{ijpu}, increments $IBEG$ by one, and repeats the calculations until the loop for group LSG_{iu} is completed. GBG_{ijpu} is then the minimum group number such that the following bounds are true for arrival time period p: $IBEG \leq p < IBEG + MRN_{iu}$. Likewise, GED_{ijpu} is the maximum group number for which the bounds are true. Figure 9 illustrates the formation of incompatible arc groups for arcs with minimum arrival and return periods using this procedure.

The next step of this procedure then determines arc incompatibility for all remaining arcs (which have return time periods greater than MRN_{iu} or arrival time periods greater than MNA_{iu}). It selects an incompatible arc by comparing the time period that the arc leaves node i and the time period in which it returns to node i with the take-off and return time periods of the incompatible arc groups formed in the first step. Once again, the procedure initializes $IBEG$ to MNB_{iu} for the first group and iterates through the loop to calculate GBG_{ijpu} and GED_{ijpu} until group number LSG_{iu} is checked. The bounds for period p are more complex in this step because the arrival time periods are not equal to MNA_{iu} or the return time periods are not equal to MRN_{iu}. GBG_{ijpu} is the



MNA = 1 MRN = 2

s_1 to d_1	GBG	GED
━━━━━━━━	1	1
••••••••••••••	1	2
┊┊┊┊┊┊┊┊┊┊┊┊┊┊	2	2
Group 1	━━━━	••••••
Group 2	•••••••	┊┊┊┊┊┊┊┊

Figure 9. Illustration of Step 1 of Generating Incompatible Arc Groups

minimum group number such that the following bounds are true for arrival time period p:

$p \geq IBEG - MNA_{i\mu} - PAR_{ij\mu}$ and

$p < IBEG + (2 \times MRN_{i\mu}) - MNA_{i\mu} + PRT_{ij\mu} - PAR_{ij\mu}$.

Likewise, $GED_{ij\mu}$ is the maximum group number for which the bounds hold true. Figure 10 illustrates this step in generating incompatible arcs groups.

Although it appears that all incompatible groups have been generated, one more step is required because in some situations two arcs in $\mathbf{IG}_g$ may be compatible. This situation occurs when the earliest arc in a group to leave for one destination is compatible with the latest arc in the same group to leave for another destination. Figure 11 illustrates the situation when two compatible arcs are in the same group. To avoid compatibility within incompatible arc groups, the compatible arc *that does not have the minimum return period* $PRT_{ij\mu}$ *and the minimum arrival period* $PAR_{ij\mu}$ is removed from that group. Removing this arc does not change its relationship with other incompatible arcs because it does not have the minimum return and arrival period and must have already been included in previous incompatible arc groups.

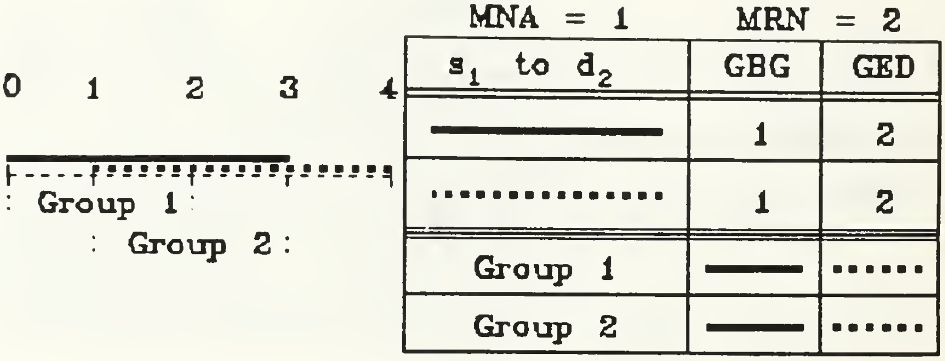


Figure 10. Illustration of Step 2 of Generating Incompatible Arc Groups

The final step of the procedure is to eliminate the arcs added to $\mathbf{IG}_g$ in the second step that are actually compatible with arcs placed in $\mathbf{IG}_g$ in the first step. Looping through all incompatible arc groups $\mathbf{IG}_g$, where $1 \leq g \leq LSG_{i_n\mu}$, change $GBG_{i_N jp\mu}$ to the maximum of $GBG_{i_N jp\mu}$ and $g + 1$ if the following is true:

- $PRT_{ij\mu} > MRN_{i\mu}$ or $PRT_{ij\mu} > MRN_{i\mu}$ and
- $p \geq MNB_{i\mu} + (g - 1) + MRN_{i\mu} - MNA_{i\mu} + PAR_{ij\mu}$.

Change $GED_{i_N jp\mu}$ to the minimum of $GED_{i_N jp\mu}$ and $g - 1$ if the following is true:

- $PRT_{ij\mu} > MRN_{i\mu}$ or $PRT_{ij\mu} > MRN_{i\mu}$ and
- $p < MNB_{i\mu} + (g - 1) + MRN_{i\mu} - MNA_{i\mu} + PAR_{ij\mu} - PRT_{ij\mu}$.

Figure 11 illustrates the final step of the procedure that generates incompatible arc groups.

11. Flow Existence Rules

Once the mine and mode networks have been generated, membership in the time-expanded network for Model A is simple to express. Rules for membership in the existence sets are as follows:

- $x_{ijp\mu} \in$ XTE if the arc from i to j via mode μ is part of the original network and $BGP_{ij\mu} \leq p \leq EDP_{ij\mu}$.

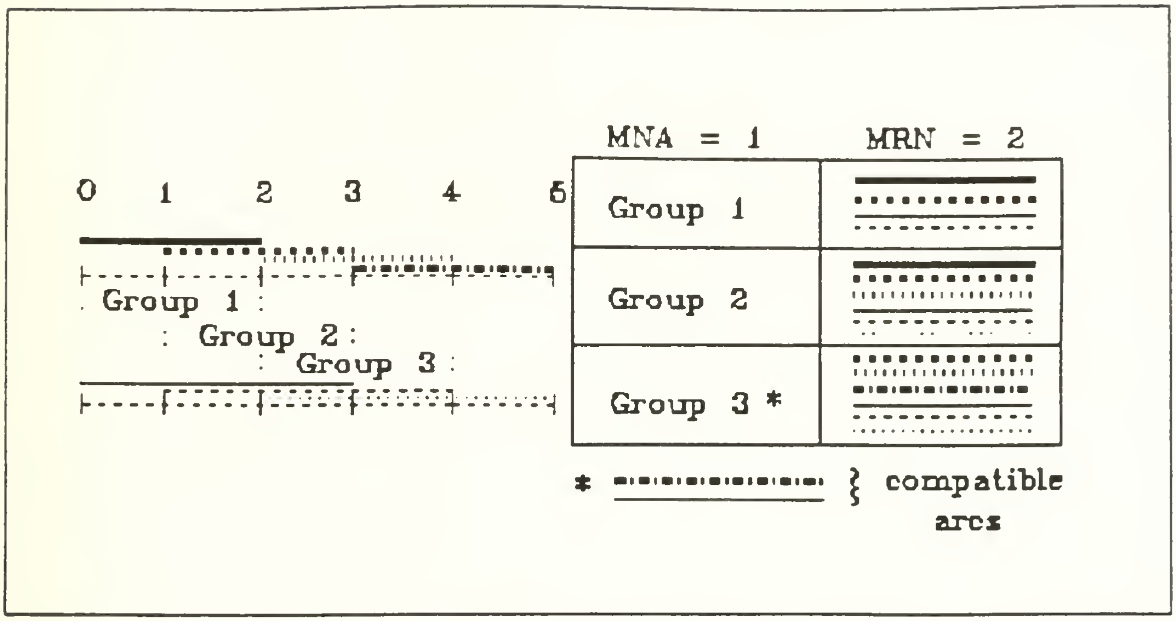


Figure 11. Illustration of the Final Step in Generating Incompatible Arc Groups

- $y_{ijp\mu} \in$ YTE if the arc from i to j via mode μ is part of the original network and $BGP_{ij\mu} \leq p \leq EDP_{ij\mu}$.

The mine and mode flow existence sets for the Model B are very similar to those of the original model:

- $x_{ijp\mu} \in$ XTE if the arc from i to j via mode μ is part of the original network and $BGP_{ij\mu} \leq p \leq EDP_{ij\mu}$.
- $y_{ijp\mu} \in$ YTE if the arc from i to j via mode μ is part of the original network and $BGP_{ij\mu} \leq p \leq EDP_{ij\mu}$.
- $y_{ijp\mu} \in \mathrm{IG}_g$ if $y_{ijp\mu} \in$ YTE and $GBG_{ijp\mu} \leq g \leq GED_{ijp\mu}$.

III. COMPUTATIONAL RESULTS

A. COMPUTER IMPLEMENTATION OF PROPOSED MODELS

GAMS, General Algebraic Modeling System, [Ref. 7] was selected for several reasons to implement the proposed models into computer code and to compare the relative efficiencies of the models. GAMS is a software package that incorporates FORTRAN-based solvers for optimization problems. The advantages of GAMS in developing optimization models are that GAMS allows changes to be made simply, takes care of mundane details such as array sizes, generates all equations needed to solve the problem based on the algebraic expressions given in the model formulation, and uses relational databases to organize the data. Also, GAMS outputs the size of the problem in terms of the number of individual constraints (called "equations" by GAMS) and the number of discrete and continuous variables as well as information on the efficiency of the model in terms of the number of iterations and the computer resources needed to solve the problem. Such output can be used to choose the fastest model of the two proposed models for SUMIT.

B. COMPARISON OF MODELS

Since Model B eliminates the need for tracking mode inventory, it should have a little over half the number of binary variables generated by Model A in most cases. Fewer binary variables should make the alternate model easier to solve and, thus, faster than the original model. To test this hypothesis, ten different problems were solved by both models. Table 1 summarizes the input data for the ten problems. Since having more than three transshipment nodes for a regional problem is rare, the number of transshipment nodes is set at two for all ten problems. Also, the number of node-mode combinations, which is the number of positive $NUM_{i\mu}$ parameters input by the user, is included rather than the number of modes. The number of node-mode combinations is more representative of the size of the model since all mode types are rarely stationed at the same origin node. To keep this thesis unclassified, the input data for the ten problems do not represent actual data, but are realistic approximations of actual data. The resulting output concerning model efficiency is expressed in Table 2. Both models arrived at the same optimal solution for all ten problems, which demonstrates the validity of Model B. The problems were executed on an Amdahl 5990 mainframe using

GAMS/ZOOM, where is ZOOM is one of the packages available with GAMS for mixed integer programs.

Table 1. INFORMATION ABOUT SCENARIOS FOR TEN TEST PROBLEMS

Model Number	End time	Number of Periods	Number of Nodes	Number of MOMAG/ Supply Nodes	Number of Demand Nodes	Number of Node-Mode Combinations
1	30	10	10	6	2	8
2	50	10	11	6	3	9
3	27	10	11	5	4	8
4	26	6	10	6	2	8
5	30	6	11	6	3	10
6	27	7	9	4	3	8
7	30	6	12	7	3	9
8	75	15	12	7	3	7
9	45	7	11	5	4	8
10	26	7	10	6	2	8
Average	37	8	11	6	3	8

Several important conclusions about the relative efficiencies of the two models can be made, based on the information presented in Table 2. For all ten problems, Model B produced fewer constraints than Model A, reducing the number of equations by an average of 15%. Both models have the same number of continuous variables because both models contain the same flow balance equations to track mine flow and mine inventory. Table 2 shows that, on average, Model B has fewer binary variables (by 39%), fewer iterations (by 87%), and smaller work parameters (by 76%). The work parameter controls the number of nodes waiting to be checked during the "branch and bound" portion of execution. If the work parameter is set too small, ZOOM will terminate execution and ask the user to reset the work parameter. Thus, for problems similar to the test problems, the sixth column of Table 2 suggests that setting the work parameter at 10,000 would suffice. In short, the data in Table 2 indicates that, for problems similar to the ten problems tested, Model B yields fewer constraints, binary variables, and iterations and smaller work parameters.

Table 2. RESULTS OF TEN TEST RUNS TO COMPARE BOTH MODELS

Problem and Model Numbers	Number of Constraints	Number of Continuous Variables	Number of Discrete Variables	Number of Iterations	Work Parameter
1A	160	102	96	167,646	15,614
1B	136	102	54	41,641	5.319
2A	237	169	144	155,958	4.568
2B	211	169	94	4,153	100
3A	179	118	114	213,304	2,361
3B	155	118	69	57,593	217
4A	75	39	43	23,663	1,403
4B	56	39	22	3,707	257
5A	144	92	90	106,720	3,283
5B	125	92	59	8,053	824
6A	137	96	84	149,176	5,634
6B	122	96	57	36.735	2,330
7A	86	50	47	187,531	4,119
7B	66	50	26	4,407	543
8A	142	100	81	318,461	5,161
8B	116	100	49	43,308	1,455
9A	143	96	87	401,748	7.525
9B	127	96	55	26,773	1,041
10A	83	45	50	150,445	3,178
10B	62	45	27	9,128	474
Average A	139	91	84	187,465	5,195
Average B	118	91	51	23,550	1.256
Percent Reduction	15%	0%	39%	87%	76%

Of the ten problems executed by Model A, problem nine had the highest number of iterations and required 12 minutes of CPU time to reach optimality. When problem nine was solved by Model B, CPU time was cut to 0.75 minutes. Problem seven was the most dramatic illustration of the reduction in CPU time, while problem three was the least dramatic. For problem seven, the CPU time for Model A was 3.5 minutes, which was reduced to only 0.1 minutes when solved by Model B. The CPU time taken by Model A and by Model B to solve problem three was 7.5 minutes and two minutes respectively. The fact that Model B produces fewer constraints and fewer binary variables means that it requires fewer iterations and a smaller work parameter to reach optimality. This reduction in size and increase in speed, *cut the CPU time by two-thirds* in the worst-case situation of the ten problems tested.

C. GAMS IMPLEMENTATION OF SUMIT

The implementation of SUMIT and a sample problem in GAMS is given in Appendix A. The output from the first run of the problem in Appendix A is contained in Appendix B. The input for and the output from the second, and final run, are contained in Appendices C and D respectively. Appendix E discusses several aspects of the GAMS implementation of SUMIT, including a description of SUMIT's output.

Because large mixed integer problems are hard to solve, the user should limit the number of time periods to ten if the problem has more than ten nodes. However, since it is difficult to judge the end time and time period length for a new problem, the user should follow the recommendations given in Appendix E to find appropriate values for the end time and time period length. Notice that the efficiency data in Table 2 does not include the initial runs discussed in Appendix E because these runs are very quick.

Other mixed integer program solvers are also available with GAMS. The solver, XA, solved the first run of sample problem in Appendix A in only 18,859 iterations, while, as shown in Appendix B, ZOOM required 76,589 iterations. Thus, XA seems to be superior to ZOOM in solving mine transshipment problems using the SUMIT model.

A way to attack large problems is to allow SUMIT to divide the large problem into several subproblems. SUMIT does not require that the mine or the mode network be connected. In other words, a path need not exist between all nodes in each network. If the network can be divided into multiple connected components, then SUMIT can split the problem into separate subproblems, one for each component. For networks that are large enough to tax the computer's disk space, running SUMIT for each connected component may solve a problem that could not otherwise be solved. By setting the scalar *DISC* equal to 0, SUMIT will run the optimization on the first connected component of the network that it finds and will notify the user of the nodes not contained in the first component. The user must then rerun SUMIT for the nodes that were not in the first connected component.

The sample program given in Appendix A is an example of a large problem that was divided into subproblems by SUMIT. Appendix B contains the first solution output by SUMIT. SUMIT tells the user to rerun the program for nodes not included in the first component found. If there are at least one supply node and one demand node in this list that have not already been included in components already solved, the user should rerun SUMIT for the nodes indicated. The user may rerun SUMIT without deleting excessive amounts of input data by setting (1) $NUM_{i\mu}$ equal to 0 for every node and mode combination that was included in the mine network for the first run of SUMIT and (2) AMT_d equal to 0 for all demand already filled by the first run. Appendix C contains the input portion of SUMIT for the second run and illustrates this simple change. Notice that, since the second problem was smaller, the end time *TEND* and time period length *TPER* could be decreased.

In many cases, finding the proven optimal solution is very expensive in computer time and memory, so GAMS provides an option that allows the user to specify an acceptable "optimality gap." During the branch and bound portion of execution, ZOOM finds an upper bound and a lower bound on the objective value for the optimal solution. ZOOM terminates execution and reports an "INTEGER SOLUTION" if it has found a feasible integer solution in which the absolute distance between these two bounds divided by the lower bound is less than the optimality gap, called OPTCR. The integer solution reported when ZOOMS halts execution for this reason is not a proven optimal solution. In fact, when running the ten test problems using Model B, the solver found the optimal solution, without proving optimality, by using the GAMS default value of OPTCR (0.1) in six cases. Optimal solutions were also found, but not proven, for three of the remaining four problems by using an OPTCR of 0.01. Appendix E contains recommendations for setting an appropriate OPTCR.

D. REDUCTION OF MINE FLOW AND INVENTORY IN BOTH MODELS

As stated previously, both models have the same number of continuous variables since they both contain the same flow balance equations for mine flow. Creating "reasonable" mine flow and inventory variables decreases the number of continuous variables needed in the model. In the computer implementation of SUMIT, applying the network generation and flow existence rules ensures that only "reasonable" variables for mine flow and inventory are used. In the first portion of the "Pre-Model Manipulation" section of the program (see Appendix A), the set parameter $ARC_{ij\mu}$ indicates the arcs that make sense for mine flow in the mine network. In the part of this section preceding the generation of incompatible arc groups, the set parameter $XTE_{ijp\mu}$ indicates the times for which mine flow associated with the arcs in $ARC_{ij\mu}$ can exist. Likewise, the set parameter MNU_{ip} shows the only time periods in which potential changes in mine inventory may occur. Tables 3 and 4 illustrate the reduction of mine flow and inventory, respectively, for the ten test problems. They compare the potential number of variables (mine flow and inventory) with the number of variables actually created by SUMIT. Using the power set **N × N × M** inflates the number of potential variables because the number of node-mode combinations can be estimated from the input data. Thus, the value for **N × M** is replaced by the number of node-mode combinations used earlier in Table 1. This swap is indicated by **N-M** in Tables 3 and 4 and provides a more accurate estimation of the number of potential variables.

Table 3. MINE FLOW REDUCTION

Model Number	Power Set N x N-M	Number of ARCs	Percent Reduction	Power Set N x N-M x P	Number of XTEs	Percent Reduction
1	80	12	85%	800	54	93%
2	99	14	86%	990	94	91%
3	88	13	85%	880	69	92%
4	80	12	85%	480	22	95%
5	110	21	81%	660	59	91%
6	72	15	79%	504	57	89%
7	108	9	92%	648	26	96%
8	84	7	92%	1260	49	96%
9	88	10	89%	616	55	91%
10	80	12	85%	560	27	95%
Average	89	13	86%	740	51	93%

The reduction in the number of variables for mine flow and inventory also affects the number of discrete variables because mine flow and mode flow are linked by the linking equations. In fact, the number of discrete variables (which is the number of variables for mode flow) created by Model B always equals the number of mine flow variables. Thus, by only including reasonable mine flow variables, SUMIT considers only logical mode flow variables. Decreasing the number of mode flow variables reduces the number of discrete variables, which increases the speed of execution for SUMIT.

Table 4. MINE INVENTORY REDUCTION

Model Number	Power Set N x N-M	Number of MNUs	Percent Reduction
1	100	47	57%
2	110	74	32%
3	110	48	56%
4	60	16	73%
5	66	32	52%
6	63	38	40%
7	72	23	68%
8	180	50	72%
9	77	40	48%
10	70	17	76%
Average	91	39	57%

IV. CONCLUSIONS

This thesis developed a single-commodity mine transshipment model, called SUMIT, for the rapid deployment of mines in a region of the world over a time window of two to three days. It considered two possible versions of SUMIT for computer implementation into GAMS. For ten problems tested, Model A and Model B arrived at the same optimal solution, but differed in relative efficiencies. For the size of the problems tested, Model B appeared to be faster and is, therefore, the model of choice for the implementation of SUMIT into GAMS.

Transshipment models involving integer programs tend to be difficult to solve, even for small problems. When limited to a region of the world spanning a few days, the mine transshipment problem is small enough to attempt integer programming. This thesis explored a viable approach for solving small mine transshipment problems using integer programs by replacing the binary mode networks, which was done in Model A, with sets of incompatible arc groups for every mode, which was done in Model B. The results of the ten test problems in this thesis demonstrate the superiority of using incompatible arc groups in the size and speed of the model for problem comparable to the test problems.

In addition to providing optimal or nearly optimal schedules for mine transshipment, SUMIT can be used as one of many tools for making decisions concerning mine warfare. SUMIT can give insight to logistical planners at CMWC in deciding the quantity and location of pre-staged mines, the quantity of mines that should be fully assembled at MOMAGs, and the MOMAGs that should receive resources for dual assembly lines to decrease the amount of time needed to deploy sets of mine fields for the most likely scenarios. In preparation for mine deployment, SUMIT can be run for different combinations of nodes and modes to indicate where modes should be based to decrease deployment time. Finally, SUMIT can be used to demonstrate the advantages of building new mine laying platforms, such as the HVMLs, by not only illustrating the time that could be saved in mine deployment for short-term regional scenarios, but also indicating which non-mine warfare platforms would be allowed to perform other critical missions.

Several enhancements could be made to SUMIT to improve its user friendliness and expand the size of the problems that it can manage:

- Develop a "front end" program in another language to read user input from a data file and printout the GAMS program for execution. The front end program would read data from GOPAS and allow users to use SUMIT without directly interfacing with GAMS.
- Develop a program in another language to read the GAMS output file and construct schedules that are more readable.
- Allow a mode group from a given node to be split between destinations when appropriate.
- Determine whether or not allowing more than two levels of mine readiness would be practical. This could be accomplished by splitting the MOMAG nodes so that each level of readiness for disassembled mines is represented by a MOMAG node and by putting one arc between MOMAG nodes for each level of readiness for disassembled mines and the associated supply node for assembled mines.
- Include constraints that take into account the limits placed on mine loading and unloading capacities at non-MOMAG supply sites and the restrictions on the number of aircraft allowed at transshipment sites at one time. Currently, this data is not available in GOPAS.

Finally, the problem discussed in this thesis could be expanded for follow-on research in two ways:

- Develop a model that encompasses world-wide scenarios of longer durations. This problem could be approached by adapting the integer rounding technique developed by Puntenney to SUMIT [Ref. 6].
- Develop a multi-commodity transshipment model by applying the work of Collier, Lally, and Puntenney [Refs. 4, 5 , 6].

APPENDIX A. SAMPLE PROGRAM, FIRST RUN

```
$TITLE  * * * Scheduler for Mine Transshipment (SUMIT) * * *
*----DOCUMENTATION-----------------------------------------------
*    Software to Optimize Schedules for Mine Transshipment
*    Developed for Commander, Mine Warfare Command
*    Written by Tammy L. Glaser, LT, USN
*    Complete documentation contained in thesis
*    06 September 1991
*----GAMS AND DOLLARS CONTROL OPTIONS----------------------------
*   Do not change the following options.
$OFFUPPER OFFSYMXREF OFFSYMLIST
OPTIONS SOLPRINT = OFF, LIMROW = 0, LIMCOL = 0;
*   Increase the following options only when recommended in the
* solution report of the output.
OPTIONS RESLIM = 10000, ITERLIM = 10000000, WORK = 10000;
*   Set OPTCR as recommended on page 72 of the thesis documentation.
OPTIONS OPTCR = 0.00001;
*----DEFINITIONS AND DATA----------------------------------------
*   See Chapter 2 of thesis documentation to explain data in detail.
SCALARS
*   The following scalars must be set to values greater than 0.
   TEND  end time of problem (h)                               /48/
   TPER  length of time period (h)                             /6/
   RELT  zero tolerance for comparisons of real data           /0.001/
*   The following scalars are optional (enter 0 for default value).
   MNLA  minimum distance for air modes over land routes (nm)    /50/
   MXSS  maximum distance for transfers between supply nodes (nm) /100/
   DISC  1 to run entire problem and 0 to split into subproblems  /0/;

SETS
*    The first two sets must have the element 0 in them.  Note that
* N must contain all elements in CN(N), found in the optional section.
* M must contain all elements in DM(M) (and TM(M), BM(M), CM(M), and
* LM(M), which are found in the optional section).  See page 7 of
* thesis documentation to explain the MOMAG-supply node split.
   N      nodes                /0, M8GQ, S8GQ, M9RP, S9RP, M10JA, S10JA,
                                M11SC, S11SC, ANDERSNAFB, NASADAK,
                                MCASIWKNI, NASJAX, CVNA, CVNB, SSN688A,
                                SSN688B, MINEFIELD1, MINEFIELD2,
                                MINEFIELD3, MINEFIELD4/
   M      modes                /0, TRUCK, C130, C141, P3, B52, TACAIR,
                                SSN688DM, DBRM/
   DM(M)  delivery modes      /P3, B52, TACAIR, SSN688DM/
*   P must start with 0 and end with the value of the number of time
* periods plus one.  G and I must start with 1.  G must end with
* the number of time periods, while I must end with the square of
* the number of nodes (nonzero elements in N).
   P      time periods        /0 * 9/
   G      group numbers       /1 * 8/
   I      iteration numbers   /1 * 400/
*   The following sets are optional.  Enter 0 between slashes for sets
* not needed for the problem.  If CN(N) is empty (i.e., initialized to
* 0), nodes are assumed to be land nodes.  If there are no transshipment
* modes or disassembled mines, enter 0 between slashes for TM(M) or
* BM(M).  All modes are assumed to be air modes if CM(M) and LM(M)
* are initialized to 0.
   CN(N)  sea nodes                /CVNA, CVNB, SSN688A, SSN688B/
   TM(M)  transportation modes     /TRUCK, C130, C141/
   BM(M)  dummy build rate modes   /DBRM/
   CM(M)  dummy sea modes          /SSN688DM/
   LM(M)  land modes               /TRUCK/
* Do not change the following evolution set.
   E      mode evolution           /TLD, TUL, TGD/;

* Do not change the following set aliases:
ALIAS (N,N1,N2,N3,N4);
ALIAS (M,M1,M2,M3);
ALIAS (P,P1);
ALIAS (I,I1);

PARAMETERS
*   The following parameter is required for node N:
   AMT(N)       supply (+) or demand (-) at node N (mines)
                /M8GQ                  200
                 S8GQ                   25
                 M9RP                  159
                 M10JA                 270
                 S10JA                  95
                 M11SC                 100
                 S11SC                  40
                 ANDERSNAFB             89
                 CVNA                  161
                 CVNB                  100
                 SSN688A                50
                 SSN688B                40
                 MINEFIELD1           -195
                 MINEFIELD2           -249
                 MINEFIELD3           -199
                 MINEFIELD4           -201/

*   The following parameters are optional (leave blank line with no
* slashes for default values):
   PRN(N)       priority of node N
                /MINEFIELD1              2
                 MINEFIELD2              1
                 MINEFIELD3              3
                 MINEFIELD4              1/

   MXD(N)       maximum distance between mine fields at node N (nm)
                /MINEFIELD2             50/

   BGD(N)       transit distance for sea node N at beginning (nm)
                /CVNA                   33
                 CVNB                   75/

   EDD(N)       transit distance for sea node N at end (nm)
                /CVNB                   75/
   SPX(N)        transit speed for sea node N (nm per h)
                 /CVNA                25
                  CVNB                25/

*   The following parameters are required for mode M:
   CAP(M)        capacity of mode unit in mode group M (mines per mode)
                 /TRUCK                20
                  C130                 20
                  C141                 20
                  P3                    8
                  B52                  22
                  TACAIR               51
                  SSN688DM             50/

   MXL(M)        maximum distance that mode M may travel (nm)
                 /TRUCK               150
                  C130               3000
                  C141               3000
                  P3                 1500
                  B52                3000
                  TACAIR              500
                  SSN688DM            200/

   SPD(M)        transit speed of mode M (nm per h)
                 /TRUCK                30
                  C130                380
                  C141                410
                  P3                  350
                  B52                 480
                  TACAIR              450
                  SSN688DM             15
                  DBRM                  9/

*  The following parameter is optional (leave blank line for default):
   PRM(M)        priority of mode M
                 /TRUCK                 1
                  C130                  3
                  C141                  3
                  P3                    1
                  B52                   1
                  TACAIR                1
                  SSN688DM              3
                  DBRM                  2/

*  The following parameter is required for mode M at node N:
   NUM(N,M)      number of mode M at node N (modes)
                 /M8GQ.DBRM             1
                  S8GQ.TRUCK            1
                  M9RP.DBRM             2
                  S9RP.C130             4
                  S9RP.P3               6
                  M10JA.DBRM            1
                  S10JA.P3              6
                  M11SC.DBRM            1
                  S11SC.TRUCK           3
                  S11SC.C130            4
                 ANDERSNAFB.B52         12
                 ANDERSNAFB.C130         4
                 NASADAK.P3              6
                 MCASIWKNI.TACAIR        1
                 NASJAX.P3               6
                 CVNA.TACAIR             1
                 CVNB.TACAIR             1
                 SSN688A.SSN688DM        1
                 SSN688B.SSN688DM        1/

*  The following parameters are optional (leave blank line for default):
   BGT(N,M)     earliest beginning time for mode M at node N (h)
                /SSN688A.SSN688DM       10/

   EDT(N,M)     latest end time for mode M at node N (h)
                /CVNA.TACAIR            24/

   MXT(N,M)     maximum total time mode M can be absent from node N (h)

   MXR(N,M)     maximum number of runs for mode M from node N
                /ANDERSNAFB.B52          2
                 ANDERSNAFB.C130         2
                 S9RP.C130               2/

   XMP(N,N1,M)  1 means eliminate runs from nodes N to N1 via mode M
                /S9RP.MINEFIELD2.P3      1/
   ;

*  The following two parameters are required.
* READ PAGE 22 OF THESIS DOCUMENTATION, WHICH EXPLAINS WHEN NEGATIVE
* DISTANCES MUST BE USED.
TABLE DIS(N,N1) distance between node N and node N1 (nm)
              M8GQ      S8GQ      M9RP      S9RP     M10JA     S10JA
M8GQ             0      -0.5      1345      1345      1157      1157
S8GQ          -0.5         0      1345      1345      1157      1157
M9RP          1345      1345         0      -0.5       738       738
S9RP          1345      1345      -0.5         0       738       738
M10JA         1157      1157       738       738         0      -0.5
S10JA         1157      1157       738       738      -0.5         0
M11SC         7501      7501      7754      7754      7028      7028
S11SC         7501      7501      7754      7754      7028      7028
ANDERSNAFB     -30       -30      1364      1364      1172      1172
NASADAK       2939      2939      3614      3614      2881      2881
MCASIWKNI     1631      1631      1821      1821      1083      1083
NASJAX        7485      7485      7768      7768      7032      7032
CVNA          1566      1566      1580      1580       849       849
CVNB          7789      7789      8020      8020      7293      7293
SSN688A       2405      2405      2881      2881      2144      2144
SSN688B       7812      7812      8125      8125      7388      7388
MINEFIELD1    2459      2459      2909      2909      2170      2170
MINEFIELD2    1819      1819      1725      1725      1015      1015
MINEFIELD3    2207      2207       897       897      1203      1203
MINEFIELD4    7797      7797      8152      8152      7414      7414

 +            M11SC  S11SC  ANDERSNAFB  NASADAK  MCASIWKNI
M8GQ           7501   7501         -30     2939       1631
```

```
S8GQ          7501    7501         -30      2939        1631
M9RP          7754    7754        1364      3614        1821
S9RP          7754    7754        1364      3614        1821
M10JA         7028    7028        1172      2881        1083
S10JA         7028    7028        1172      2881        1083
M11SC            0    -0.5        7520      4222        5942
S11SC         -0.5       0        7520      4222        5942
ANDERSNAFB    7520    7520           0      2939        1653
NASADAK       4222    4222        2939         0        1824
MCASIWKNI     5942    5942        1653      1824           0
NASJAX        -124    -124        7500      4209        5950
CVNA          6175    6175        1583      2088         267
CVNB           294     294        7801      4511        5942
SSN688A       4899    4899        2421       777        1067
SSN688B        420     420        7831      4546        6305
MINEFIELD1     4866   4866        2455       744        2053
MINEFIELD2     6032   6032        1800      2032         352
MINEFIELD3     7607   7607        2188      3850        2053
MINEFIELD4      548    548        7812      4551        6305

 +            NASJAX    CVNA     CVNB      SSN688A  SSN688B
M8GQ            7485    1566     7789         2405     7812
S8GQ            7485    1566     7789         2405     7812
M9RP            7768    1580     8020         2881     8152
S9RP            7768    1580     8020         2881     8152
M10JA           7032     849     7293         2144     7414
S10JA           7032     849     7293         2144     7414
M11SC           -124    6175      294         4899      420
S11SC           -124    6175      294         4899      420
ANDERSNAFB      7500    1583     7801         2421     7831
NASADAK         4209    2088     7801          777     4546
MCASIWKNI       5950     267     6216         1067     6305
NASJAX             0    6188      351         4900      366
CVNA            6188       0     6445         1324     6546
CVNB             366    6445        0         5164      327
SSN688A         4900    1324     5164            0     5249
SSN688B          366    6546      327         5249        0
MINEFIELD1      4869    1344     5146          110     5220
MINEFIELD2      6051     234     6294         1244     6413
MINEFIELD3      7660    1786     7817         3073     8023
MINEFIELD4       463    6578      500         5270      174

 +            MINEFIELD1    MINEFIELD2    MINEFIELD3    MINEFIELD4
M8GQ                2459          1819          2207          7797
S8GQ                2459          1819          2207          7797
M9RP                2909          1725           897          8152
S9RP                2909          1725           897          8152
M10JA               2170          1015          1203          7414
S10JA               2170          1015          1203          7414
M11SC               4866          6032          7607           548
S11SC               4866          6032          7607           548
ANDERSNAFB          2455          1800          2188          7812
NASADAK              744          2032          3850          4551
MCASIWKNI           1090           352          2053          6332
NASJAX              4869          6051          7660           463
CVNA                1344           234          1786          6578
```

```
CVNB                 5146          6294          7817           500
SSN688A               110          1244          3073          5270
SSN688B              5220          6413          8023           174
MINEFIELD1              0          1265          3081          5243
MINEFIELD2           1265             0          1817          6459
MINEFIELD3           3081          1817             0          8109
MINEFIELD4           5243          6459          8109             0;

TABLE TEV(M,E) time needed to complete evolution E for mode M (h)
            TLD         TUL          TGD
TRUCK      1.00        2.00         1.00
C130       4.00        2.00         8.00
C141       4.00        2.00         8.00
P3         1.00        0.25         4.00
B52        4.00        0.50         8.00
TACAIR     1.00        0.25         2.00
SSN688DM   0.00        1.50         0.00;

**************************************************************************
*                                                                        *
*   Input is complete.  Do not change any statements after this line     *
*                                                                        *
**************************************************************************
*----PRE-MODEL MANIPULATIONS----------------------------------------------
SCALAR
   NRLT  negative zero tolerance for comparisons of real data
   STOP  stopping criteria for loop that finds connected component
   SUMA  number of arcs in the mine network
   IBEG  counter that is incremented in incompatible arc group loop;

SETS
   NO(N)          dummy node                    /0/
   MN(N)          momag nodes
   SN(N)          supply nodes
   TN(N)          transshipment nodes
   DN(N)          demand nodes
   LN(N)          land nodes
   MO(M)          dummy mode                    /0/
   AM(M)          air modes
   ARC(N,N1,M)    arcs in network from node N to node N1 via mode M
   ART(N,N1,M)    temporary version of ARC
   DNO(N)         arcs in first connected component found
   NFC(N)         nodes that do not form a complete component
   XTE(N,N1,P,M)  arcs in time-expanded network for period P
   MNU(N,P)       mine flow balance equation used for N at period P;

*   Verify correctness of the inputted data.
ABORT$(RELT LE 0) "ERROR:  RELT must be positive.", RELT;
ABORT$(RELT GE 0.00100001) "ERROR:  RELT must be less than 0.001.", RELT;
ABORT$(TEND LE RELT) "ERROR:  TEND must be positive.", TEND;
ABORT$(TPER LE RELT) "ERROR:  TPER must be positive.", TPER;
ABORT$(TPER GE (TEND + RELT)) "ERROR:  TPER must be less than TEND.",
   TPER, TEND;
ABORT$((MNLA + RELT) LE 0) "ERROR:  MNLA must be nonnegative.", MNLA;
ABORT$((MXSS + RELT) LE 0) "ERROR:  MXSS must be nonnegative.", MXSS;
ABORT$((ABS(DISC - 1) GE RELT) AND (ABS(DISC) GE RELT))
```

```
   "ERROR:  DISC must be equal to 0 or 1.", DISC;

*   Initialize or set default values for certain parameters.
TEND = FLOOR(TEND / TPER);
NRLT = -1 * RELT;
MXSS$(MXSS LE NRLT) = SMAX((N,N1), ABS(DIS(N,N1))) + (2 * RELT);
AMT(N) = FLOOR(AMT(N))$(AMT(N) GE 0.5) + CEIL(AMT(N))$(AMT(N) LE -0.5);
MXL(M)$BM(M) = -0.5;
NUM(N,M) = FLOOR(NUM(N,M));
XMP(N,N1,M)$(NO(N) * NO(N) * MO(M)) = 0;
DIS(N,N1)$(NO(N) * NO(N1)) = 0;
DIS(N,N1)$(ABS(DIS(N,N1)) LE RELT) = DIS(N1,N);
TM(M)$MO(M) = NO;
BM(M)$MO(M) = NO;
CN(N)$NO(N) = NO;
CM(M)$MO(M) = NO;
LM(M)$MO(M) = NO;

*   Verify the correctness of the inputted data.
ABORT$(SUM(M$DM(M), 1) LE 0.5) "ERROR:  No delivery modes entered", DM;
ABORT$(SUM(N$(AMT(N) GE 0.5), AMT(N)) LE 0.5)
   "ERROR:  No supply amounts (+) entered.", AMT;
ABORT$(SUM(N$(AMT(N) LE -0.5), ABS(AMT(N))) LE 0.5)
   "ERROR:  No demand amounts (-) entered.", AMT;
ABORT$(SUM(N, AMT(N)) LE -0.5)
   "ERROR:  Total demand (-) exceeds total supply (+).", AMT;
ABORT$(SMIN(M$(NOT (MO(M) + BM(M))), MXL(M)) LE RELT)
   "ERROR:  All MXL(M) must be positive.  "
   "Did you forget the maximum range for dummy sea modes?", MXL, CM;
ABORT$(SMIN(M$(NOT (MO(M) + BM(M))), SPD(M)) LE RELT)
   "ERROR:  All SPD(M) must be positive.  "
   "Did you forget the maximum range for dummy sea modes?", SPD, CM;
ABORT$(SMIN((N,M), NUM(N,M)) LE -0.5)
   "ERROR:  NUM(N,M) must all be nonnegative.", NUM;
ABORT$(SUM((N,N1), ABS(DIS(N,N1))) LE RELT)
   "ERROR:  No distances (DIS(N,N1)) inputted", DIS;
ABORT$(SUM((N,N1,M)$((ABS(XMP(N,N1,M) - 1) GE RELT) AND
   (ABS(XMP(N,N1,M)) GE RELT)), 1) GE 0.5)
   "ERROR:  All XMP(N,N1,M) must be equal to 0 or 1.", XMP;

*   Categorize nodes and modes into appropriate subsets.
MN(N) = YES$((AMT(N) GE 0.5) AND (SUM(M$BM(M), NUM(N,M)) GE 0.5));
SN(N) = YES$((NOT (NO(N) + MN(N)))$((AMT(N) GE 0.5)
   OR ((ABS(AMT(N)) LE 0.5) AND (SUM(N1$MN(N1),
   1$(ABS(DIS(N,N1) + 0.5) LE RELT)) GE 0.5))));
TN(N) = YES$((NOT (NO(N) + SN(N)))$(ABS(AMT(N)) LE 0.5));
DN(N) = YES$(AMT(N) LE  -0.5);
CN(N) = YES$(CN(N) + DN(N));
LN(N) = YES$(NOT (CN(N) + NO(N)));
AM(M) = YES$(NOT (LM(M) + CM(M) + BM(M) + MO(M)));

*   Include all logical arcs in network.
ARC(N,N1,M)$(((MN(N) + SN(N) + TN(N)) * (SN(N1) + TN(N1) + DN(N1)))
   $((NUM(N,M) GE 0.5) AND (ABS(ORD(N) - ORD(N1)) GE 0.5)
   AND ((ABS(MXL(M)) - ABS(DIS(N,N1))) GE NRLT)
   AND (ABS(DIS(N,N1)) GE RELT))) = YES;
*   Eliminate all arcs as indicated by user.
ARC(N,N1,M)$(ABS(XMP(N,N1,M) - 1) LE RELT) = NO;
*   Eliminate all arcs between supply nodes on land via land modes
* that have nonnegative distances or have distances greater than MXSS.
ARC(N,N1,M)$((SN(N) * SN(N1) * LN(N) * LN(N1) * TM(M) * LM(M))
   $((DIS(N,N1) GE NRLT) OR ((MXSS - ABS(DIS(N,N1))) LE RELT))) = NO;
*   Eliminate all arcs between supply nodes on land via air trans-
* portation when the arcs are over land and absolute distances are
* less than MNLA or their absolute distances are greater than MXSS.
ARC(N,N1,M)$((SN(N) * SN(N1) * LN(N) * LN(N1) * TM(M) * AM(M))
   $((((MNLA - ABS(DIS(N,N1))) GE RELT) AND (DIS(N,N1) LE NRLT))
   OR ((MXSS - ABS(DIS(N,N1))) LE RELT))) = NO;
*   Eliminate all arcs between two supply nodes in which at least
* one node is on land or that use nontransportation modes.
ARC(N,N1,M)$(SN(N) * SN(N1) * (NOT (LN(N) * LN(N1) * TM(M)))) = NO;
*   Eliminate all arcs between nodes via land transportation modes
* for which the distance is nonnegative.
ARC(N,N1,M)$((TM(M) * LM(M))$(DIS(N,N1) GE NRLT)) = NO;
*   Eliminate all arcs between land nodes via air transportation
* modes when the arcs are over land and absolute distances are
* less than MNLA.
ARC(N,N1,M)$((LN(N) * LN(N1) * TM(M) * AM(M))
   $(((MNLA - ABS(DIS(N,N1))) GE RELT) AND (DIS(N,N1) LE NRLT))) = NO;
*   Eliminate all arcs from sea nodes to nondemand nodes.
ARC(N,N1,M)$(CN(N) * (NOT DN(N1))) = NO;
*   Eliminate all arcs to either supply or transshipment nodes
* via delivery modes.
ARC(N,N1,M)$((SN(N1) + TN(N1)) * DM(M)) = NO;
*   Eliminate all arcs to demand nodes via nondelivery modes.
ARC(N,N1,M)$(DN(N1) * (NOT DM(M))) = NO;
*   Eliminate all arcs leaving MOMAG nodes to nodes for which
* the distance is not equal to -0.5.
ARC(N,N1,M)$(MN(N)$(ABS(DIS(N,N1) + 0.5) GE RELT)) = NO;
*   Eliminate all arcs arriving at MOMAG nodes.
ARC(N,N1,M)$MN(N1) = NO;
*   Eliminate all arcs from transshipment nodes to supply nodes.
ARC(N,N1,M)$(TN(N) * SN(N1)) = NO;

*   Verify that only one arc leaves each MOMAG node.
ABORT$(SMAX(N$MN(N), SUM((N1,M)$ARC(N,N1,M), 1)) GE 1.5)
   "ERROR:  At least one MOMAG node has more than one arc leaving it.  ",
   "MOMAG nodes and the current network is listed as follows:", MN, ARC;

PARAMETERS
   PAR(N,N1,M)    time periods to travel from N to N1 via M one-way
   PRT(N,N1,M)    time periods to travel from N to N1 via M round trip
   IND(N)         in degree of (number of arcs entering) node N
   OTD(N)         out degree of (number of arcs leaving) node N
   BGP(N,N1,M)    earliest time to include arc from N to N1 via M
   EDP(N,N1,M)    latest time to include arc from N to N1 via M
   MXM(N)         maximum number of mines through N to fill demands
   MXS(N)         maximum number of mines through N from supply nodes
   ZUP(N)         upper bound on mine inventory at N
   XUP(N,N1,M)    upper bound on mine flow from N to N1 via M
   RHX(N,P)       fixed value on mine inventory at N at period P
   MRN(N,M)      minimum return period PRT from N via M
   MRX(N,M)      maximum return period PRT from N via M
   MNA(N,M)      minimum arrival period PAR from N via M if PRT=MRN
   MNB(N,M)      minimum begin period BGP from N via M
   MXE(N,M)      maximum end period EDP from N via M
   LSG(N,M)      last group number needed from N via M
   GBG(N,N1,P,M) first group number from N to N1 via M at P
   GED(N,N1,P,M) last group number from N to N1 via M at P;

*   Initialize or set default values for certain parameters.
* Convert times to time periods.
CAP(M) = FLOOR(CAP(M));
PRN(N)$NO(N) = 0;
PRM(M)$MO(M) = 0;
PRN(N)$((NOT NO(N))$(PRN(N) LE RELT)) = MAX(1, SMAX(N1, PRN(N1)));
PRM(M)$((NOT MO(M))$(PRM(M) LE RELT)) = MAX(1, SMAX(M1, PRM(M1)));
MXD(N)$(NOT DN(N)) = 0;
BGD(N)$NO(N) = 0;
EDD(N)$NO(N) = 0;
SPX(N)$NO(N) = 0;
BGT(N,M)$(NO(N)*MO(M)) = 0;
EDT(N,M)$(NO(N)*MO(M)) = 0;
SUMA = 0;
MXT(N,M)$(NO(N)*MO(M)) = 0;
MXR(N,M)$(NO(N)*MO(M)) = 0;
GBG(N,N1,P,M)$(NO(N) * NO(N1) * MO(M)) = 0;
GED(N,N1,P,M)$(NO(N) * NO(N1) * MO(M)) = 0;
MXR(N,M)$(MXR(N,M) GE RELT) = FLOOR(MXR(N,M));
MXT(N,M)$(MXT(N,M) GE RELT) = FLOOR(MXT(N,M) / TPER);
MXT(N,M)$((NOT (NO(N) + MO(M)))
   $((MXT(N,M) LE RELT) AND (NUM(N,M) GE 0.5))) = TEND;
EDT(N,M)$((NOT (NO(N) + MO(M)))
   $((EDT(N,M) LE RELT) AND (NUM(N,M) GE 0.5))) = TPER * TEND;

*   Calculate capacity for dummy build rate modes.
CAP(M)$BM(M) = FLOOR(SPD(M) * TPER);

*   Verify the correctness of the inputted data.
STOP = TEND + 1;
ABORT$((ABS(CARD(P) - 2 - TEND)) GE RELT)
   "ERROR:  The set P must contain integers from 0 to 'STOP'",
   "(see the next line for the value of 'STOP').", STOP, P;
ABORT$(ABS(CARD(G) - TEND) GE RELT)
   "ERROR:  The set G must contain integers from 1 to 'TEND'.", TEND, G;
STOP = (CARD(N) - 1) ** 2;
ABORT$(ABS(CARD(I) - STOP) GE RELT)
   "ERROR:  The set I must contain integers from 1 to 'STOP'",
   "(see the next line for the value of 'STOP').", STOP, I;
ABORT$((SMIN(N, PRN(N)) + RELT) LE 0)
   "ERROR:  PRN(N) must all be nonnegative.", PRN;
ABORT$((SMIN(N, MXD(N)) + RELT) LE 0)
   "ERROR:  MXD(N) must all be nonnegative.", MXD;
ABORT$((SMIN(N, BGD(N)) + RELT) LE 0)
   "ERROR:  BGD(N) must all be nonnegative.", BGD;
ABORT$((SMIN(N, EDD(N)) + RELT) LE 0)
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   "ERROR:  EDD(N) must all be nonnegative.", EDD;
ABORT$((SMIN(N, SPX(N)) + RELT) LE 0)
   "ERROR:  SPX(N) must all be nonnegative.", SPX;
ABORT$(SUM(N$(((BGD(N) GE RELT) OR (EDD(N) GE RELT))
   AND (SPX(N) LE RELT)), 1) GE 0.5)
   "ERROR:  If BGD(N) or EDD(N) is inputted, SPX(N) must be positive.",
   BGD, EDD, SPX;
ABORT$(SMIN(M$(NOT MO(M)), CAP(M)) LE 0.5)
   "ERROR:  CAP(M) must all be positive.  ",
   "Did you forget the capacity for dummy sea modes?", CAP, CM;
ABORT$((SMIN(M, PRM(M)) + RELT) LE 0)
   "ERROR:  PRM(M) must all be nonnegative.", PRM;
ABORT$((SMIN((N,M), BGT(N,M)) + RELT) LE 0)
   "ERROR:  BGT(N,M) must all be nonnegative.", BGT;
ABORT$((SMIN((N,M), EDT(N,M)) + RELT) LE 0)
   "ERROR:  EDT(N,M) must all be nonnegative.", EDT;
ABORT$((SMIN((N,M), MXT(N,M)) + RELT) LE 0)
   "ERROR:  MXT(N,M) must all be nonnegative.", MXT;
ABORT$((SMIN((N,M), MXR(N,M)) + RELT) LE 0)
   "ERROR:  MXR(N,M) must all be nonnegative.", MXR;
ABORT$((SMIN(N, SPX(N)) + RELT) LE 0)
   "ERROR:  SPX(N) must all be nonnegative.", SPX;
ABORT$((SMIN((M,E), TEV(M,E)) + RELT) LE 0)
   "ERROR:  TEV(M,E) must all be nonnegative.", TEV;

*   Adjust BGT and EDT if nonzero BGD and EDD is inputted.  Convert
* BGT and EDT to time periods.
BGT(N,M)$(NOT (NO(N) + MO(M)))
   = CEIL(((BGD(N) / SPX(N))$((SPX(N) GE RELT) AND (NUM(N,M) GE 0.5))
   + BGT(N,M)) / TPER);
EDT(N,M)$(NOT (NO(N) + MO(M)))
   = FLOOR(((EDD(N) / SPX(N))$((SPX(N) GE RELT) AND (NUM(N,M) GE 0.5))
   + EDT(N,M)) / TPER);

*   Calculate PAR taking into account load, travel, and unload/
* delivery times.  Calculate PRT taking into account load,
* travel, unload/delivery, return travel, and ground times.
PAR(N,N1,M)$(ARC(N,N1,M)$(SPD(M) GE RELT))
   = CEIL(((TEV(M,'TLD') + TEV(M,'TUL')) / TPER) + 1$BM(M)
   +  ((ABS(DIS(N,N1)) + MXD(N1)) / (TPER * SPD(M)))$(NOT BM(M)));
PRT(N,N1,M)$(ARC(N,N1,M)$(SPD(M) GE RELT))
   = CEIL(((TEV(M,'TLD') + TEV(M,'TUL') + TEV(M,'TGD')) / TPER)
   + (((2 * ABS(DIS(N,N1))) + MXD(N1)) / (TPER * SPD(M)))$(NOT BM(M))
   + 1$BM(M));

*   Eliminate all arcs from N to N1 via M if PRT is greater than
* MXT or (EDT - BGT).  If node N has no supply and no incoming arcs,
* eliminate all arcs directed from node N.
ARC(N,N1,M)$(ARC(N,N1,M)$(((MXT(N,M) - PRT(N,N1,M)) LE NRLT)
   OR ((PRT(N,N1,M) - (EDT(N,M) - BGT(N,M))) GE RELT))) = NO;
ARC(N,N1,M)$((SUM((N2,M1)$ARC(N2,N,M1), 1) LE 0.5)
    AND (ABS(AMT(N)) LE 0.5)) = NO;
ART(N,N1,M) = ARC(N,N1,M);

*  In a loop, take the following steps until a loop in which
* no arcs are deleted from the mine network is made:
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```
*     Calculate SUMA, IND, and OTD.  Initialize DNO and MXM.
*     Calculate EDP and MXM by checking all possible paths
*     in the network.  Reinitialize DNO and MXS.  Calculate
*     BGP and MXS by checking all possible paths in the network.
*     (In these loops, DNO indicates whether or not MXM or MXS
*     has already been updated for origin node of the arc being
*     checked.)  Calculate EDP for arcs leaving MOMAG nodes.
*     Eliminate arcs from N to N1 via M for which PRT is
*     greater than (EDP - BGP).
*     If node N has no supply and no incoming arcs from
*     other nodes, eliminate all arcs directed from node N.
*     If DISC equals 0, then set
*     DNO to 'yes' for all nodes in the first connected
*     component found.  If DISC equals 1, then set DNO to 'yes'
*     for all nodes (except dummy node 0).  Eliminate arcs whose
*     nodes are not contained in the first component found.
LOOP(I1$(ABS(SUMA - SUM((N,N1,M)$ART(N,N1,M), 1)) GE RELT),
   SUMA = SUM((N,N1,M)$ART(N,N1,M), 1);
   IND(N) = SUM((N1,M)$ART(N1,N,M), 1);
   OTD(N) = SUM((N1,M)$ART(N,N1,M), 1);
   DNO(N) = YES;
   MXM(N) = 0;
   LOOP(N$(DN(N)$((IND(N) + OTD(N)) GE 0.5)),
      DNO(N) = NO;
      LOOP((N1,M)$ART(N1,N,M),
         EDP(N1,N,M) = MIN(EDT(N1,M), TEND) - PRT(N1,N,M) + PAR(N1,N,M);
         MXM(N1)$(DNO(N1) + DNO(N)) = ABS(AMT(N)) + MXM(N1);
         DNO(N1) = NO;
         LOOP((N2,M1)$ART(N2,N1,M1),
            EDP(N2,N1,M1) = MAX(EDP(N2,N1,M1),
               MIN(EDP(N1,N,M) - PAR(N1,N,M), MIN(EDT(N2,M1),TEND)
               - PRT(N2,N1,M1) + PAR(N2,N1,M1)));
            MXM(N2)$(DNO(N2) + DNO(N)) = ABS(AMT(N)) + MXM(N2);
            DNO(N2) = NO;
            LOOP((N3,M2)$ART(N3,N2,M2),
               EDP(N3,N2,M2) = MAX(EDP(N3,N2,M2),
                  MIN(EDP(N2,N1,M1) - PAR(N2,N1,M1),
                  MIN(EDT(N3,M2), TEND)
                  - PRT(N3,N2,M2) + PAR(N3,N2,M2)));
               MXM(N3)$(DNO(N3) + DNO(N)) = ABS(AMT(N)) + MXM(N3);
               DNO(N3) = NO;
               LOOP((N4,M3)$ART(N4,N3,M3),
                  EDP(N4,N3,M3) = MAX(EDP(N4,N3,M3),
                     MIN(EDP(N3,N2,M2) - PAR(N3,N2,M2),
                     MIN(EDT(N4,M3), TEND)
                     - PRT(N4,N3,M3) + PAR(N4,N3,M3)));
                  MXM(N4)$(DNO(N4) + DNO(N)) = ABS(AMT(N)) + MXM(N4);
                  DNO(N4) = NO))));
      DNO(N1)$(NOT DN(N1)) = YES);
   BGP(N,N1,M)$ART(N,N1,M)
      = PAR(N,N1,M)$(IND(N) LE 0.5) + TEND$(IND(N) GE 0.5)
      + BGT(N,M);
   DNO(N) = YES;
   MXS(N) = 0;
   LOOP(N$((IND(N) LE 0.5) AND (OTD(N) GE 0.5)),
      MXS(N)$DNO(N) = AMT(N);
      DNO(N) = NO;
      LOOP((N1,M)$ART(N,N1,M),
         MXS(N1)$(DNO(N) + DNO(N1)) = MXS(N1) + AMT(N)
            + AMT(N1)$(NOT DN(N1));
         DNO(N1) = NO;
         LOOP((N2,M1)$ART(N1,N2,M1),
            BGP(N1,N2,M1) = MIN(BGP(N1,N2,M1), PAR(N1,N2,M1)
               + BGP(N,N1,M)$(AMT(N1) LE 0.5) + BGT(N1,M1));
            MXS(N2)$(DNO(N) + DNO(N2)) = MXS(N2) + AMT(N)
               + AMT(N1)$(NOT DN(N1)) + AMT(N2)$(NOT DN(N2));
            DNO(N2) = NO;
            LOOP((N3,M2)$ART(N2,N3,M2),
               BGP(N2,N3,M2) = MIN(BGP(N2,N3,M2), PAR(N2,N3,M2)
                  + BGP(N1,N2,M1)$(AMT(N2) LE 0.5) + BGT(N2,M2));
               MXS(N3)$(DNO(N) + DNO(N3)) = MXS(N3) + AMT(N)
                  + AMT(N1)$(NOT DN(N1)) + AMT(N2)$(NOT DN(N2))
                  + AMT(N3)$(NOT DN(N3));
               DNO(N3) = NO;
               LOOP((N4,M3)$ART(N3,N4,M3),
                  BGP(N3,N4,M3) = MIN(BGP(N3,N4,M3), PAR(N3,N4,M3)
                     + BGP(N2,N3,M2)$(AMT(N3) LE 0.5) + BGT(N3,M3));
                  MXS(N4)$(DNO(N) + DNO(N4)) = MXS(N4) + AMT(N)
                     + AMT(N1)$(NOT DN(N1)) + AMT(N2)$(NOT DN(N2))
                     + AMT(N3)$(NOT DN(N3)) + AMT(N4)$(NOT DN(N4));
                  DNO(N4) = NO))));
      DNO(N1)$(IND(N1) GE 0.5) = YES);
   EDP(N,N1,M)$(MN(N)$((CAP(M) * NUM(N,M)) GE RELT)) = MIN(EDP(N,N1,M),
      CEIL(MIN(MXS(N),MXM(N)) / (CAP(M) * NUM(N,M))));
   ART(N,N1,M)$(ART(N,N1,M)$((EDP(N,N1,M) - BGP(N,N1,M)) LE NRLT)) = NO;
   ART(N,N1,M)$((SUM((N2,M1)$ART(N2,N,M1), 1) LE 0.5)
      AND (ABS(AMT(N)) LE 0.5)) = NO;
*   Initialize DIS, DNO and STOP if DISC equals 0.
   DIS(N,N1)$(DISC LE 0.5) = 0;
   LOOP(M$(DISC LE 0.5),
      DIS(N,N1)$ART(N,N1,M) = 1);
   DNO(N)$(DISC LE 0.5)
      = YES$(ABS(SMIN((N1,N2,M)$((NOT NO(N1)) * ART(N1,N2,M)), ORD(N1))
      - ORD(N)) LE RELT);
   STOP$(DISC LE 0.5) = SUM((N,N1)$DNO(N), DIS(N,N1) + DIS(N1,N));
*   Starting with the first nondummy node in network found, set
* DNO to 'yes' for any node which (1) is on a path with the first
* node found or (2) is on a backtrack path from other nodes on a
* path with the first node found.  Stop when all arcs in the network
* have been checked.
   LOOP(I$((STOP GE 0.5) AND (DISC LE 0.5)),
      LOOP(N$DNO(N),
         LOOP(N1$((DIS(N,N1) GE 0.5) OR (DIS(N1,N) GE 0.5)),
            DNO(N1) = YES;
            DIS(N,N1)$(DIS(N,N1) GE 0.5) = 0;
            DIS(N1,N)$(DIS(N1,N) GE 0.5) = 0));
      STOP = SUM((N,N1)$DNO(N), DIS(N,N1) + DIS(N1,N)));
   DNO(N)$((NOT NO(N))$(ABS(DISC - 1) LE RELT))  = YES;
   ART(N,N1,M)$(NOT (DNO(N) * DNO(N1))) = NO;
*   Ensure that the first component found contains both supply and
* demand.  If not, repeat the process for the next component.
   SUMA$(((SUM(N$(DNO(N) * DN(N)), ABS(AMT(N))) LE 0.5)
      OR (SUM(N$(DNO(N) * SN(N)), AMT(N)) LE 0.5))$(DISC LE 0.5)) = 0;
   NFC(N)$(DNO(N)$((SUM(N1$(DNO(N1) * SN(N1)), AMT(N1)) LE 0.5)
      OR (SUM(N2$(DNO(N2) * DN(N2)), ABS(AMT(N2))) LE 0.5))) = YES;
   ART(N,N1,M)$((NOT (NFC(N) * NFC(N1))) * ARC(N,N1,M)) = YES);

*   Ensure that total supply is greater than total demand
* in generated network.
ABORT$((SUM(N$(DNO(N) * DN(N)), ABS(AMT(N)))
   - SUM(N$(DNO(N) * (MN(N) + SN(N))), AMT(N))) GE 0.5)
   "ERROR:  Total demand (-) exceeds total supply (+) ",
   "in generated network.", DNO, AMT;

*   Reset the following parameters if N is not in the first connected
* component found.
ART(N,N1,M) = ARC(N,N1,M);
ARC(N,N1,M)$(NOT (DNO(N) * DNO(N1))) = NO;
MXT(N,M)$(NOT DNO(N)) = 0;
MXR(N,M)$(NOT DNO(N)) = 0;
BGT(N,M)$(NOT DNO(N)) = 0;
EDT(N,M)$(NOT DNO(N)) = 0;
PAR(N,N1,M)$(NOT ARC(N,N1,M)) = 0;
PRT(N,N1,M)$(NOT ARC(N,N1,M)) = 0;
BGP(N,N1,M)$(NOT ARC(N,N1,M)) = 0;
EDP(N,N1,M)$(NOT ARC(N,N1,M)) = 0;
NUM(N,M)$(SUM(N1$PRT(N,N1,M), 1) LE 0.5) = -1 * NUM(N,M);

*   Set XTE equal to 'yes' for all arcs from N to N1 via M that
* are part of the mine network if P is less than or equal to EDP
* and greater than or equal to BGP.  Set MNU to 'yes' if at least
* one arc exists from N to N1 (or from N1 to N) via mode M at time
* period P.
XTE(N,N1,P,M) = YES$(ARC(N,N1,M)$(((ORD(P) - (EDP(N,N1,M) + 1)) LE RELT)
   AND ((ORD(P) - BGP(N,N1,M)) GE RELT)));
MNU(N,P)$(NOT NO(N)) = YES
   $(SUM((N1,M), 1$(XTE(N1,N,P,M) + XTE(N,N1,P + PAR(N,N1,M),M))) GE 0.5);

*   Calculate ZUP, XUP, and RHX using AMT, MXS and MXM.
ZUP(N)$(NOT NO(N)) = ABS(AMT(N))$DN(N) + MIN(MXS(N), MXM(N))$(NOT DN(N));
XUP(N,N1,M)$ARC(N,N1,M) = MIN(ZUP(N), ZUP(N1), CAP(M) * NUM(N,M));
RHX(N,P)$(DNO(N)$(ABS(ORD(P) - SMIN(P1$MNU(N,P1), ORD(P1))) LE RELT))
   = ABS(AMT(N)) - 2$(ABS(AMT(N)) LE RELT);
RHX(N,P)$(DNO(N)
   $(ABS(ORD(P) - 1 - SMAX(P1$MNU(N,P1), ORD(P1))) LE RELT)) = -1;
RHX(N,P)$(DNO(N)$(RHX(N,P) GE RELT)) = MIN(RHX(N,P), ZUP(N));

*   Calculate the following parameters in preparation for forming
* incompatible arc groups.
MRN(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMIN(N1$ARC(N,N1,M), PRT(N,N1,M));
MRX(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMAX(N1$ARC(N,N1,M), PRT(N,N1,M));
MNA(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMIN(N1$(ARC(N,N1,M)
   $(ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)), PAR(N,N1,M));
MNB(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMIN(N1$ARC(N,N1,M), BGP(N,N1,M));
MXE(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMAX(N1$ARC(N,N1,M), EDP(N,N1,M));
LSG(N,M)$(DNO(N)$(NUM(N,M) GE 0.5)) = SMAX(N1$(ARC(N,N1,M)
   $((ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)
   AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT))), EDP(N,N1,M));
LSG(N,M)$(DNO(N)$(LSG(N,M) GE RELT)) = 2 + LSG(N,M) - MNB(N,M) - MRN(N,M);
LSG(N,M)$(DNO(N)$(((MRN(N,M) - (MXE(N,M) - MNB(N,M))) GE RELT)
   AND (SUM(N1$ARC(N,N1,M), 1) GE 1.5))) = 1;

*   Calculate GBG and GED.  This loop is explained in depth on page
* 27 of the thesis documentation.
LOOP((N,N1,M)$((DNO(N) * (NOT MN(N)))$((NUM(N,M) GE 0.5)
   AND ((MRX(N,M) - 1 GE RELT) OR ((ABS(MRX(N,M) - 1) LE RELT)
   AND (SUM(N2$ARC(N,N2,M), 1) GE 1.5))))),
   LOOP(P$XTE(N,N1,P,M),
      IBEG = MNB(N,M);
      LOOP(G$((LSG(N,M) - ORD(G)) GE NRLT),
         GBG(N,N1,P,M)$((GBG(N,N1,P,M) LE RELT)
            AND (ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)
            AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT))
            = ORD(G)$(((ORD(P) - 1 - IBEG) GE NRLT)
            AND ((IBEG + MRN(N,M) - ORD(P) + 1) GE RELT));
         GED(N,N1,P,M)$((ABS(PRT(N,N1,M) - MRN(N,M)) LE RELT)
            AND (ABS(PAR(N,N1,M) - MNA(N,M)) LE RELT))
            = MAX(ORD(G)$((ABS(IBEG + MRN(N,M) - ORD(P) + 1) GE RELT)
            AND ((ORD(P) - 1 - IBEG) GE NRLT)), GED(N,N1,P,M));
         GBG(N,N1,P,M)$((GBG(N,N1,P,M) LE RELT)
            AND ((ABS(PRT(N,N1,M) - MRN(N,M)) GE RELT)
            OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT)))
            = ORD(G)$(((IBEG + 2 * MRN(N,M) - MNA(N,M)
            - ORD(P) + PRT(N,N1,M) - PAR(N,N1,M)) GE NRLT)
            AND ((ORD(P) - 1 + PAR(N,N1,M)
            - IBEG + MNA(N,M)) GE NRLT));
         GED(N,N1,P,M)$((ABS(PRT(N,N1,M) - MRN(N,M)) GE RELT)
            OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
            = MAX(ORD(G)$(((IBEG + 2 * MRN(N,M) - MNA(N,M)
            - ORD(P) + PRT(N,N1,M) - PAR(N,N1,M)) GE NRLT)
            AND ((ORD(P) - 1 + PAR(N,N1,M)
            - IBEG + MNA(N,M)) GE NRLT)), GED(N,N1,P,M));
            IBEG = IBEG + 1)));

LOOP((G,N,M)$((LSG(N,M) - ORD(G)) GE NRLT),
   LOOP((N1,P)$((GBG(N,N1,P,M) GE RELT) AND ((ABS(PRT(N,N1,M)
      - MRN(N,M)) GE RELT) OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
      AND ((ORD(P) - PAR(N,N1,M) - MNB(N,M) - ORD(G) - MRN(N,M)
      + MNA(N,M)) GE NRLT)),
      GBG(N,N1,P,M) = MAX(GBG(N,N1,P,M), ORD(G) + 1)));

LOOP((G,N,M)$((LSG(N,M) - ORD(G)) GE NRLT),
   LOOP((N1,P)$((GED(N,N1,P,M) GE RELT) AND ((ABS(PRT(N,N1,M)
      - MRN(N,M)) GE RELT) OR (ABS(PAR(N,N1,M) - MNA(N,M)) GE RELT))
      AND ((MNB(N,M) + ORD(G) + MRN(N,M) - MNA(N,M)
      - ORD(P)- PRT(N,N1,M) + PAR(N,N1,M)) GE RELT)),
      GED(N,N1,P,M) = MIN(GED(N,N1,P,M), ORD(G) - 1)));
*----MODEL FOR SUMIT--------------------------------------------------
POSITIVE VARIABLES
   X(N,N1,P,M)  flow of mines from N to N1 via mode M at time period P
   Z(N,P)       mines inventory at N at time period P;
```

```
* fix upper bounds on positive variables; set some positive inventory
*   variables to be constant
X.UP(N,N1,P,M)$XTE(N,N1,P,M) = XUP(N,N1,M);
Z.UP(N,P) = ZUP(N);

FREE VARIABLE
   V            objective value;

BINARY VARIABLE
   Y(N,N1,P,M)  1 (0) if path (not) used from N to N1 via M at P;

EQUATIONS
   OBJV              objective value
   MNFB(N,P)         flow balance equations for mine network
   LINK(N,N1,P,M)    link equations between mine and mode networks
   MDSK(N,M,G)       mode schedule equations for incompatible arcs
   MDPG(N,M)         constraint to limit total time that M is from N
   MDRG(N,M)         constraint to limit total runs that M is from N;

OBJV..
   V =G= SUM((N,N1,P,M)$XTE(N,N1,P,M), (PRN(N) * PRN(N1) * PRM(M)
     * (ORD(P) - 1 + PRT(N,N1,M))) ** 2 * Y(N,N1,P,M));

MNFB(N,P)$(MNU(N,P) * DNO(N))..
  0 =G= (1$DN(N) - 1$(NOT DN(N)))
    * (Z(N,P)$((RHX(N,P) LE 0.5) AND (RHX(N,P) GE -1.5))
    - Z(N,P + 1)$(RHX(N,P + 1) GE -0.5) + RHX(N,P)$(RHX(N,P) GE 0.5))
    - SUM((N1,M), X(N1,N,P,M)$XTE(N1,N,P,M)
    - X(N,N1,P + PAR(N,N1,M),M)$XTE(N,N1,P + PAR(N,N1,M),M));

LINK(N,N1,P,M)$XTE(N,N1,P,M)..
   0 =L= XUP(N,N1,M) * Y(N,N1,P,M) - X(N,N1,P,M);

MDSK(N,M,G)$(DNO(N)$((LSG(N,M) - ORD(G)) GE NRLT))..
   1 =G= SUM((N1,P)$(((ORD(G) - GBG(N,N1,P,M)) GE NRLT)
   AND ((GED(N,N1,P,M) - ORD(G)) GE NRLT)), Y(N,N1,P,M));

MDPG(N,M)$((DNO(N) * (NOT (NO(N) * MO(M))))$((TEND - MXT(N,M)) GE RELT))..
   SUM((N1,P)$XTE(N,N1,P,M), PRT(N,N1,M) * Y(N,N1,P,M)) =L= MXT(N,M);

MDRG(N,M)$((DNO(N) * (NOT (NO(N) * MO(M))))$(MXR(N,M) GE RELT))..
   SUM((N1,P)$XTE(N,N1,P,M), Y(N,N1,P,M)) =L= MXR(N,M);

MODEL SUMIT /OBJV,MNFB,MDSK,LINK,MDPG,MDRG/;

SOLVE SUMIT USING MIP MINIMIZING V;

*-----SOLUTION REPORT------------------------------------------------
DISPLAY "   The following arcs were considered by SUMIT.", ARC;

DISPLAY "   *** All times in given in hours, incremented by TPER. ***";

PARAMETER
   TIMECOMP(N,*)  Time in which laying of mine field N was completed;
   TIMECOMP(N,'TIME.DONE')$(DN(N) * DNO(N))
      = SMAX((N1,P,M)$(Y.L(N1,N,P,M) GE RELT), TPER * (ORD(P) - 1));
   OPTION TIMECOMP:1;
   DISPLAY TIMECOMP;

PARAMETER
   MOMAGREP(N,*)  MOMAG Schedule Report    (Start Time is 0);
   MOMAGREP(N,'START.TIME')$((MN(N) * DNO(N))
      $(SMAX((N1,P,M)$XTE(N,N1,P,M), X.L(N,N1,P,M)) GE RELT))
      = TPER * SMIN((N1,P,M)$(X.L(N,N1,P,M) GE RELT), ORD(P) - 2);
   MOMAGREP(N,'STOP.TIME')$((MN(N) * DNO(N))
      $(SMAX((N1,P,M)$XTE(N,N1,P,M), X.L(N,N1,P,M)) GE RELT))
      = TPER * SMAX((N1,P,M)$(X.L(N,N1,P,M) GE RELT), ORD(P) - 1);
   MOMAGREP(N,'NUM.MINES')$(MN(N) * DNO(N))
      = SUM((N1,P,M)$(X.L(N,N1,P,M) GE RELT), X.L(N,N1,P,M));
   OPTION MOMAGREP:1:1:1;
   DISPLAY MOMAGREP;

PARAMETER
   SCHEDULE(N,M,G,N1,*)  Schedule for Mine Transshipment by Origin Base;
   LOOP(N$((DNO(N) * (NOT MN(N)))
      $(SUM((N1,P,M), X.L(N,N1,P,M)) GE RELT)),
      LOOP(M$(NUM(N,M) GE 0.5),
         IBEG = 1;
         LOOP((P,N1)$(X.L(N,N1,P,M) GE RELT),
            LOOP(G$(ABS(ORD(G) - IBEG) LE RELT),
               SCHEDULE(N,M,G,N1,'NO.MODES')
                  = CEIL(X.L(N,N1,P,M) / CAP(M));
               SCHEDULE(N,M,G,N1,'NO.MINES') = X.L(N,N1,P,M);
               SCHEDULE(N,M,G,N1,'LOAD.TIME')
                  = TPER * (ORD(P) - 1 - PAR(N,N1,M));
               SCHEDULE(N,M,G,N1,'ETD.ORIG')
                  = (TPER * (ORD(P) - 1 - PAR(N,N1,M))) + TEV(M,'TLD');
               SCHEDULE(N,M,G,N1,'ETA.DEST')
                  = (TPER * (ORD(P) - 1)) - TEV(M,'TUL');
               SCHEDULE(N,M,G,N1,'AVAIL.TIME')
                  = TPER * (ORD(P) - 1 + PRT(N,N1,M) - PAR(N,N1,M)));
            IBEG = IBEG + 1));
      OPTION SCHEDULE:1:4:1;
      DISPLAY SCHEDULE;
      SCHEDULE(N,M,G,N1,'NO.MODES') = 0;
      SCHEDULE(N,M,G,N1,'NO.MINES') = 0;
      SCHEDULE(N,M,G,N1,'LOAD.TIME') = 0;
      SCHEDULE(N,M,G,N1,'ETD.ORIG') = 0;
      SCHEDULE(N,M,G,N1,'ETA.DEST') = 0;
      SCHEDULE(N,M,G,N1,'AVAIL.TIME') = 0);

PARAMETER
   MINEINV(N,*)  The number of mines in left in inventory;
   MINEINV(N,'MINES.LEFT')$(DNO(N) * (MN(N) + SN(N)))
      = SUM((N1,P,M)$((X.L(N,N1,P,M) GE 0.5) OR (X.L(N1,N,P,M) GE 0.5)),
      X.L(N1,N,P,M) - X.L(N,N1,P,M)) + AMT(N);
   OPTION MINEINV:0;
   DISPLAY MINEINV;

SET
   NEXTRUN(N,*)  Nodes that should be included in the next run of SUMIT;
   NEXTRUN(N,'NEXT.RUN')$((NOT (NO(N) + DNO(N) + NFC(N)))
   $((SMAX((N1,M)$ART(N,N1,M), ABS(NUM(N,M))) GE 0.5)
   OR (AMT(N) LE -0.5))) = YES;
LOOP(I$((ABS(ORD(I) - 1) LE 0.5)
   AND (SUM(N$(NEXTRUN(N,'NEXT.RUN') * DN(N)), AMT(N)) LE -0.5)
   AND (SUM(N$NEXTRUN(N,'NEXT.RUN'), 1) GE 0.5)),
   DISPLAY "   Change NUM(N,M) and DN(N) to 0 for all nodes not in ",
      " NEXTRUN(N) and rerun SUMIT to finish the solution.", NEXTRUN;
   LOOP(I1$((ABS(ORD(I1) - 1) LE 0.5)
      AND (SUM(N$NEXTRUN(N,'NEXT.RUN'), AMT(N)) LE -0.5)),
      AMT(N) = AMT(N)$NEXTRUN(N,'NEXT.RUN');
      OPTION AMT:0;
      DISPLAY "WARNING:  There is not enough supply to meet demand ",
         "for the next run of SUMIT.", AMT));
```

APPENDIX B. SAMPLE OUTPUT, FIRST RUN

```
COMPILATION TIME      =        0.950 SECONDS



MODEL STATISTICS

BLOCKS OF EQUATIONS     6     SINGLE EQUATIONS     173
BLOCKS OF VARIABLES     4     SINGLE VARIABLES     213
NON ZERO ELEMENTS     625     DISCRETE VARIABLES    78

GENERATION TIME       =        1.190 SECONDS


EXECUTION TIME        =        7.500 SECONDS



               S O L V E     S U M M A R Y

     MODEL   SUMIT                OBJECTIVE  V
     TYPE    MIP                  DIRECTION  MINIMIZE
     SOLVER  ZOOM                 FROM LINE  821

**** SOLVER STATUS      1 NORMAL COMPLETION
**** MODEL STATUS       1 OPTIMAL
**** OBJECTIVE VALUE            45612.0000

 RESOURCE USAGE, LIMIT        158.146    10000.000
 ITERATION COUNT, LIMIT     76589     10000000

 Z O O M / X M P    ---    Version 2.1 Jun 1988

 Courtesy of Dr Roy E. Marsten,
             Department of Management Information Systems,
             University of Arizona,
             Tucson     Arizona  85721,  U.S.A.


 No options file found - using defaults.

 Work space needed (estimate)   --   67966 words.
 Work space available           --   67966 words.
 Maximum obtainable             --  294142 words.

 The LU factors occupied   804 slots  (estimate  3068).

 The branch and bound tree contained  4550 nodes (max.  10000 nodes).

 Iterations: Initial LP          193, Time:     0.26
```

```
             Heuristic          3214,           8.21
             Branch and bound  73061,         149.45
             Final LP            121,           0.14


**** REPORT SUMMARY :         0     NONOPT
                              0 INFEASIBLE
                              0  UNBOUNDED




----    815    THE FOLLOWING ARCS WERE CONSIDERED BY SUMIT.


----    815 SET       ARC           ARCS IN NETWORK FROM NODE N TO NODE
                                    N1 VIA MODE M

                       TRUCK  C130    P3  B52  TACAIR  SSN688DM  DBRM

M8GQ      .S8GQ                                                   YES
S8GQ      .ANDERSNAFB    YES
M9RP      .S9RP                                                   YES
S9RP      .MCASIWKNI           YES
S9RP      .MINEFIELD3                YES
M10JA     .S10JA                                                  YES
S10JA     .MINEFIELD2                YES
S10JA     .MINEFIELD3                YES
ANDERSNAFB.NASADAK             YES
ANDERSNAFB.MCASIWKNI           YES
ANDERSNAFB.MINEFIELD1                     YES
ANDERSNAFB.MINEFIELD2                     YES
ANDERSNAFB.MINEFIELD3                     YES
NASADAK   .MINEFIELD1                YES
MCASIWKNI .MINEFIELD2                             YES
CVNA      .MINEFIELD2                             YES
SSN688A   .MINEFIELD1                                       YES


----    817    *** ALL TIMES IN GIVEN IN HOURS, INCREMENTED BY TPER.  ***


----    824 PARAMETER TIMECOMP      TIME IN WHICH LAYING OF MINE FIELD
                                    N WAS COMPLETED

             TIME.DONE

MINEFIELD1        36.0
MINEFIELD2        36.0
MINEFIELD3        36.0


----    837 PARAMETER MOMAGREP      MOMAG SCHEDULE REPORT
                                    (START TIME IS 0)

        STOP.TIME   NUM.MINES
```

```
M8GQ          6.0         54.0
M9RP         12.0        128.0
M10JA         6.0         54.0


----    860 PARAMETER SCHEDULE      SCHEDULE FOR MINE TRANSSHIPMENT
                                    BY ORIGIN BASE
S8GQ.TRUCK
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.ANDERSNAFB     1.0      19.0                  1.0       4.0         6.0
2.ANDERSNAFB     1.0      20.0        6.0       7.0      10.0        12.0
3.ANDERSNAFB     1.0      20.0       12.0      13.0      16.0        18.0
4.ANDERSNAFB     1.0      20.0       18.0      19.0      22.0        24.0


----    860 PARAMETER SCHEDULE      SCHEDULE FOR MINE TRANSSHIPMENT
                                    BY ORIGIN BASE
S9RP.P3
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD3     6.0      48.0        6.0       7.0      11.7        18.0
2.MINEFIELD3     6.0      48.0       18.0      19.0      23.7        30.0
3.MINEFIELD3     4.0      32.0       30.0      31.0      35.7        42.0


----    860 PARAMETER SCHEDULE      SCHEDULE FOR MINE TRANSSHIPMENT
                                    BY ORIGIN BASE
S10JA.P3
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD3     6.0      48.0                  1.0       5.7        18.0
2.MINEFIELD2     6.0      48.0       18.0      19.0      23.7        30.0
3.MINEFIELD2     6.0      48.0       30.0      31.0      35.7        42.0


----    860 PARAMETER SCHEDULE      SCHEDULE FOR MINE TRANSSHIPMENT
                                    BY ORIGIN BASE
ANDERSNAFB.B52
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD3     2.0      23.0                  4.0      11.5        24.0
2.MINEFIELD1     7.0     145.0       24.0      28.0      35.5        48.0


----    860 PARAMETER SCHEDULE      SCHEDULE FOR MINE TRANSSHIPMENT
                                    BY ORIGIN BASE
CVNA.TACAIR
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD2     1.0      51.0        6.0       7.0      11.7        12.0
2.MINEFIELD2     1.0      51.0       12.0      13.0      17.7        18.0
3.MINEFIELD2     1.0      51.0       18.0      19.0      23.7        24.0
```

```
----    860 PARAMETER SCHEDULE        SCHEDULE FOR MINE TRANSSHIPMENT
                                      BY ORIGIN BASE
SSN688A.SSN688DM
            NO.MODES   NO.MINES   LOAD.TIME   ETD.ORIG   ETA.DEST   AVAIL.TIME

1.MINEFIELD1     1.0       50.0        12.0       12.0       22.5         30.0


----    874 PARAMETER MINEINV         THE NUMBER OF MINES IN LEFT
                                      IN INVENTORY

       MINES.LEFT

M8GQ          146
M9RP           31
M10JA         216
S10JA           5
CVNA            8


----    883    CHANGE NUM(N,M) AND DN(N) TO 0 FOR ALL NODES NOT IN
             NEXTRUN(N) AND RERUN SUMIT TO FINISH THE SOLUTION.


----    883 SET       NEXTRUN         NODES THAT SHOULD BE INCLUDED
                                      IN THE NEXT RUN OF SUMIT

              NEXT.RUN

M11SC              YES
S11SC              YES
NASJAX             YES
CVNB               YES
SSN688B            YES
MINEFIELD4         YES


**** FILE SUMMARY FOR USER 8693P

INPUT     TEST1    GAMS     A
OUTPUT    TEST1    LISTING  A

EXECUTION TIME       =        1.370 SECONDS
```

APPENDIX C. SAMPLE PROGRAM, SECOND RUN

```
$TITLE  * * * Scheduler for Mine Transshipment (SUMIT) * * *
*----DOCUMENTATION------------------------------------------------------
*    Software to Optimize Schedules for Mine Transshipment
*    Developed for Commander, Mine Warfare Command
*    Written by Tammy L. Glaser, LT, USN
*    Complete documentation contained in thesis
*    06 September 1991
*----GAMS AND DOLLARS CONTROL OPTIONS-----------------------------------
*   Do not change the following options.
$OFFUPPER OFFSYMXREF OFFSYMLIST
OPTIONS SOLPRINT = OFF, LIMROW = 0, LIMCOL = 0;
*   Increase the following options only when recommended in the
* solution report of the output.
OPTIONS RESLIM = 10000, ITERLIM = 10000000, WORK = 10000;
*   Set OPTCR as recommended on page 72 of the thesis documentation.
OPTIONS OPTCR = 0.00001;
*----DEFINITIONS AND DATA-----------------------------------------------
*   See Chapter 2 of thesis documentation to explain data in detail.
SCALARS
*   The following scalars must be set to values greater than 0.
   TEND  end time of problem (h)                                  /27.0/
   TPER  length of time period (h)                                /1.2/
   RELT  zero tolerance for comparisons of real data              /0.001/
*   The following scalars are optional (enter 0 for default value).
   MNLA  minimum distance for air modes over land routes (nm)     /50/
   MXSS  maximum distance for transfers between supply nodes (nm) /100/
   DISC  1 to run entire problem and 0 to split into subproblems  /0/;

SETS
*    The first two sets must have the element 0 in them.  Note that
* N must contain all elements in CN(N), found in the optional section.
* M must contain all elements in DM(M) (and TM(M), BM(M), CM(M), and
* LM(M), which are found in the optional section).  See page 7 of
* thesis documentation to explain the MOMAG-supply node split.
   N      nodes              /0, M8GQ, S8GQ, M9RP, S9RP, M10JA, S10JA,
                              M11SC, S11SC, ANDERSNAFB, NASADAK,
                              MCASIWKNI, NASJAX, CVNA, CVNB, SSN688A,
                              SSN688B, MINEFIELD1, MINEFIELD2,
                              MINEFIELD3, MINEFIELD4/
   M      modes              /0, TRUCK, C130, C141, P3, B52, TACAIR,
                              SSN688DM, DBRM/
   DM(M)  delivery modes     /P3, B52, TACAIR, SSN688DM/
*   P must start with 0 and end with the value of the number of time
* periods plus one.  G and I must start with 1.  G must end with
* the number of time periods, while I must end with the square of
* the number of nodes (nonzero elements in N).
   P      time periods       /0 * 23/
   G      group numbers      /1 * 22/
   I      iteration numbers  /1 * 400/
*   The following sets are optional.  Enter 0 between slashes for sets
* not needed for the problem.  If CN(N) is empty (i.e., initialized to
* 0), nodes are assumed to be land nodes.  If there are no transshipment
* modes or disassembled mines, enter 0 between slashes for TM(M) or
* BM(M).  All modes are assumed to be air modes if CM(M) and LM(M)
* are initialized to 0.
   CN(N)  sea nodes                /CVNA, CVNB, SSN688A, SSN688B/
   TM(M)  transportation modes     /TRUCK, C130, C141/
   BM(M)  dummy build rate modes   /DBRM/
   CM(M)  dummy sea modes          /SSN688DM/
   LM(M)  land modes               /TRUCK/
* Do not change the following evolution set.
   E      mode evolution           /TLD, TUL, TGD/;

* Do not change the following set aliases:
ALIAS (N,N1,N2,N3,N4);
ALIAS (M,M1,M2,M3);
ALIAS (P,P1);
ALIAS (I,I1);

PARAMETERS
*   The following parameter is required for node N:
   AMT(N)       supply (+) or demand (-) at node N (mines)
                /M8GQ                200
                 S8GQ                 25
                 M9RP                159
                 M10JA               270
                 S10JA                95
                 M11SC               100
                 S11SC                40
                 ANDERSNAFB           89
                 CVNA                161
                 CVNB                100
                 SSN688A              50
                 SSN688B              40
                 MINEFIELD1            0
                 MINEFIELD2            0
                 MINEFIELD3            0
                 MINEFIELD4         -201/

*   The following parameters are optional (leave blank line with no
* slashes for default values):
   PRN(N)       priority of node N
                /MINEFIELD1            2
                 MINEFIELD2            1
                 MINEFIELD3            3
                 MINEFIELD4            1/

   MXD(N)       maximum distance between mine fields at node N (nm)
                /MINEFIELD2           50/

   BGD(N)       transit distance for sea node N at beginning (nm)
                /CVNA                 33
                 CVNB                 75/

   EDD(N)       transit distance for sea node N at end (nm)
                /CVNB                 75/
   SPX(N)       transit speed for sea node N (nm per h)
                /CVNA                25
                 CVNB                25/

*   The following parameters are required for mode M:
   CAP(M)       capacity of mode unit in mode group M (mines per mode)
                /TRUCK                20
                 C130                 20
                 C141                 20
                 P3                    8
                 B52                  22
                 TACAIR               51
                 SSN688DM             50/

   MXL(M)       maximum distance that mode M may travel (nm)
                /TRUCK               150
                 C130               3000
                 C141               3000
                 P3                 1500
                 B52                3000
                 TACAIR              500
                 SSN688DM            200/

   SPD(M)       transit speed of mode M (nm per h)
                /TRUCK                30
                 C130                380
                 C141                410
                 P3                  350
                 B52                 480
                 TACAIR              450
                 SSN688DM             15
                 DBRM                  9/

*  The following parameter is optional (leave blank line for default):
   PRM(M)       priority of mode M
                /TRUCK                 1
                 C130                  3
                 C141                  3
                 P3                    1
                 B52                   1
                 TACAIR                1
                 SSN688DM              3
                 DBRM                  2/

*  The following parameter is required for mode M at node N:
   NUM(N,M)     number of mode M at node N (modes)
                /M8GQ.DBRM             0
                 S8GQ.TRUCK            0
                 M9RP.DBRM             0
                 S9RP.C130             0
                 S9RP.P3               0
                 M10JA.DBRM            0
                 S10JA.P3              0
                 M11SC.DBRM            1
                 S11SC.TRUCK           3
                 S11SC.C130            4
```

```
                    ANDERSNAFB.B52          0
                    ANDERSNAFB.C130         0
                    NASADAK.P3              0
                    MCASIWKNI.TACAIR        0
                    NASJAX.P3               6
                    CVNA.TACAIR             0
                    CVNB.TACAIR             1
                    SSN688A.SSN688DM        0
                    SSN688B.SSN688DM        1/

*   The following parameters are optional (leave blank line for default):
   BGT(N,M)     earliest beginning time for mode M at node N (h)
                /SSN688A.SSN688DM       10/

   EDT(N,M)     latest end time for mode M at node N (h)
                /CVNA.TACAIR           24/

   MXT(N,M)     maximum total time mode M can be absent from node N (h)

   MXR(N,M)     maximum number of runs for mode M from node N
                /ANDERSNAFB.B52         2
                 ANDERSNAFB.C130        2
                 S9RP.C130              2/

   XMP(N,N1,M)  1 means eliminate runs from nodes N to N1 via mode M
                /S9RP.MINEFIELD2.P3     1/
   ;

*   The following two parameters are required.
* READ PAGE 22 OF THESIS DOCUMENTATION, WHICH EXPLAINS WHEN NEGATIVE
* DISTANCES MUST BE USED.
TABLE DIS(N,N1) distance between node N and node N1 (nm)
               M8GQ      S8GQ      M9RP      S9RP     M10JA     S10JA
M8GQ              0      -0.5      1345      1345      1157      1157
S8GQ           -0.5         0      1345      1345      1157      1157
M9RP           1345      1345         0      -0.5       738       738
S9RP           1345      1345      -0.5         0       738       738
M10JA          1157      1157       738       738         0      -0.5
S10JA          1157      1157       738       738      -0.5         0
M11SC          7501      7501      7754      7754      7028      7028
S11SC          7501      7501      7754      7754      7028      7028
ANDERSNAFB      -30       -30      1364      1364      1172      1172
NASADAK        2939      2939      3614      3614      2881      2881
MCASIWKNI      1631      1631      1821      1821      1083      1083
NASJAX         7485      7485      7768      7768      7032      7032
CVNA           1566      1566      1580      1580       849       849
CVNB           7789      7789      8020      8020      7293      7293
SSN688A        2405      2405      2881      2881      2144      2144
SSN688B        7812      7812      8125      8125      7388      7388
MINEFIELD1     2459      2459      2909      2909      2170      2170
MINEFIELD2     1819      1819      1725      1725      1015      1015
MINEFIELD3     2207      2207       897       897      1203      1203
MINEFIELD4     7797      7797      8152      8152      7414      7414

 +             M11SC  S11SC  ANDERSNAFB  NASADAK  MCASIWKNI
M8GQ            7501   7501         -30     2939       1631
```

S8GQ	7501	7501	-30	2939	1631
M9RP	7754	7754	1364	3614	1821
S9RP	7754	7754	1364	3614	1821
M10JA	7028	7028	1172	2881	1083
S10JA	7028	7028	1172	2881	1083
M11SC	0	-0.5	7520	4222	5942
S11SC	-0.5	0	7520	4222	5942
ANDERSNAFB	7520	7520	0	2939	1653
NASADAK	4222	4222	2939	0	1824
MCASIWKNI	5942	5942	1653	1824	0
NASJAX	-124	-124	7500	4209	5950
CVNA	6175	6175	1583	2088	267
CVNB	294	294	7801	4511	5942
SSN688A	4899	4899	2421	777	1067
SSN688B	420	420	7831	4546	6305
MINEFIELD1	4866	4866	2455	744	2053
MINEFIELD2	6032	6032	1800	2032	352
MINEFIELD3	7607	7607	2188	3850	2053
MINEFIELD4	548	548	7812	4551	6305

+	NASJAX	CVNA	CVNB	SSN688A	SSN688B
M8GQ	7485	1566	7789	2405	7812
S8GQ	7485	1566	7789	2405	7812
M9RP	7768	1580	8020	2881	8152
S9RP	7768	1580	8020	2881	8152
M10JA	7032	849	7293	2144	7414
S10JA	7032	849	7293	2144	7414
M11SC	-124	6175	294	4899	420
S11SC	-124	6175	294	4899	420
ANDERSNAFB	7500	1583	7801	2421	7831
NASADAK	4209	2088	7801	777	4546
MCASIWKNI	5950	267	6216	1067	6305
NASJAX	0	6188	351	4900	366
CVNA	6188	0	6445	1324	6546
CVNB	366	6445	0	5164	327
SSN688A	4900	1324	5164	0	5249
SSN688B	366	6546	327	5249	0
MINEFIELD1	4869	1344	5146	110	5220
MINEFIELD2	6051	234	6294	1244	6413
MINEFIELD3	7660	1786	7817	3073	8023
MINEFIELD4	463	6578	500	5270	174

+	MINEFIELD1	MINEFIELD2	MINEFIELD3	MINEFIELD4
M8GQ	2459	1819	2207	7797
S8GQ	2459	1819	2207	7797
M9RP	2909	1725	897	8152
S9RP	2909	1725	897	8152
M10JA	2170	1015	1203	7414
S10JA	2170	1015	1203	7414
M11SC	4866	6032	7607	548
S11SC	4866	6032	7607	548
ANDERSNAFB	2455	1800	2188	7812
NASADAK	744	2032	3850	4551
MCASIWKNI	1090	352	2053	6332
NASJAX	4869	6051	7660	463
CVNA	1344	234	1786	6578

```
CVNB                 5146          6294          7817           500
SSN688A               110          1244          3073          5270
SSN688B              5220          6413          8023           174
MINEFIELD1              0          1265          3081          5243
MINEFIELD2           1265             0          1817          6459
MINEFIELD3           3081          1817             0          8109
MINEFIELD4           5243          6459          8109             0;

TABLE TEV(M,E) time needed to complete evolution E for mode M (h)
            TLD         TUL          TGD
TRUCK      1.00        2.00         1.00
C130       4.00        2.00         8.00
C141       4.00        2.00         8.00
P3         1.00        0.25         4.00
B52        4.00        0.50         8.00
TACAIR     1.00        0.25         2.00
SSN688DM   0.00        1.50         0.00;

************************************************************************
*                                                                      *
*   Input is complete.  Do not change any statements after this line.  *
*                                                                      *
************************************************************************
```

APPENDIX D. SAMPLE OUTPUT, SECOND RUN

```
COMPILATION TIME      =        0.950 SECONDS



MODEL STATISTICS

BLOCKS OF EQUATIONS     6     SINGLE EQUATIONS     123
BLOCKS OF VARIABLES     4     SINGLE VARIABLES     154
NON ZERO ELEMENTS     446     DISCRETE VARIABLES    51

GENERATION TIME       =        1.170 SECONDS


EXECUTION TIME        =        7.440 SECONDS



               S O L V E     S U M M A R Y

     MODEL   SUMIT                  OBJECTIVE  V
     TYPE    MIP                    DIRECTION  MINIMIZE
     SOLVER  ZOOM                   FROM LINE  821

**** SOLVER STATUS      1 NORMAL COMPLETION
**** MODEL STATUS       1 OPTIMAL
**** OBJECTIVE VALUE            158139.0000

 RESOURCE USAGE, LIMIT          105.820     10000.000
 ITERATION COUNT, LIMIT       66392      10000000

 Z O O M / X M P    ---    Version 2.1 Jun 1988

 Courtesy of Dr Roy E. Marsten,
             Department of Management Information Systems,
             University of Arizona,
             Tucson     Arizona  85721,  U.S.A.


 No options file found - using defaults.

 Work space needed (estimate)   --   62593 words.
 Work space available           --   62593 words.
 Maximum obtainable             --  246526 words.

 The LU factors occupied   552 slots  (estimate  2101).

 The branch and bound tree contained  4106 nodes (max.  10000 nodes).

 Iterations: Initial LP          118, Time:     0.12
```

```
           Heuristic            253,          0.62
           Branch and bound   65947,        104.94
           Final LP              74,          0.07


**** REPORT SUMMARY :          0     NONOPT
                               0 INFEASIBLE
                               0  UNBOUNDED



----    815    THE FOLLOWING ARCS WERE CONSIDERED BY SUMIT.


----    815 SET       ARC            ARCS IN NETWORK FROM NODE N TO
                                     NODE N1 VIA MODE M

                   TRUCK  C130   P3  TACAIR  SSN688DM  DBRM

M11SC  .S11SC                                            YES
S11SC  .NASJAX       YES   YES
NASJAX .MINEFIELD4               YES
CVNB   .MINEFIELD4                       YES
SSN688B.MINEFIELD4                                  YES


----    817    *** ALL TIMES IN GIVEN IN HOURS, INCREMENTED BY TPER. ***


----    824 PARAMETER TIMECOMP       TIME IN WHICH LAYING OF MINE FIELD N
                                     WAS COMPLETED

           TIME.DONE

MINEFIELD4        19.2


----    837 PARAMETER MOMAGREP       MOMAG SCHEDULE REPORT
                                     (START TIME IS 0)

        STOP.TIME   NUM.MINES

M11SC        3.6         21.0


----    860 PARAMETER SCHEDULE       SCHEDULE FOR MINE TRANSSHIPMENT
                                     BY ORIGIN BASE
S11SC.TRUCK
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.NASJAX         2.0      40.0                  1.0       5.2        13.2
2.NASJAX         2.0      21.0        3.6       4.6       8.8        16.8


----    860 PARAMETER SCHEDULE       SCHEDULE FOR MINE TRANSSHIPMENT
```

```
                                            BY ORIGIN BASE
NASJAX.P3
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD4     5.0      40.0        7.2       8.2      10.5        15.6
2.MINEFIELD4     3.0      21.0       15.6      16.6      18.9        24.0


----     860 PARAMETER SCHEDULE             SCHEDULE FOR MINE TRANSSHIPMENT
                                            BY ORIGIN BASE
CVNB.TACAIR
            NO.MODES  NO.MINES  LOAD.TIME  ETD.ORIG  ETA.DEST  AVAIL.TIME

1.MINEFIELD4     1.0      51.0        3.6       4.6       5.7         9.6
2.MINEFIELD4     1.0      49.0        9.6      10.6      11.7        15.6


----     860 PARAMETER SCHEDULE             SCHEDULE FOR MINE TRANSSHIPMENT
                                            BY ORIGIN BASE
SSN688B.SSN688DM
            NO.MODES  NO.MINES  ETA.DEST  AVAIL.TIME

1.MINEFIELD4     1.0      40.0      11.7        25.2


----     874 PARAMETER MINEINV              THE NUMBER OF MINES IN LEFT
                                            IN INVENTORY

       MINES.LEFT

M11SC          79


**** FILE SUMMARY FOR USER 8693P

INPUT     TEST2    GAMS     A
OUTPUT    TEST2    LISTING  A

EXECUTION TIME      =        1.140 SECONDS
```

APPENDIX E. GAMS IMPLEMENTATION OF SUMIT

A. SUMIT OUTPUT

The output of SUMIT is composed of standard GAMS reports and of data displayed by the program. The standard GAMS reports include an echo print of the program, compilation time, model statistics (i.e., number of constraints and continuous and discrete variables), and solve summary. The echo print of the program was deleted from Appendix B since the program is given in Appendix A. The data output by SUMIT was also edited to make Appendix B fit within the page margins. The data displayed by the program include the arcs generated for the mine network (which has the same arcs as the mode network), the time in which the mine field deployment was complete, the MOMAG schedule report, the schedules for mine transshipment, the mine inventory at the end of the problem, and the nodes that should be included in the next run of SUMIT. SUMIT prints out the mine network to allow the user to verify that all arcs in the mine and mode networks are appropriate. The MOMAG schedule report contains the times in which each MOMAG must start and stop assembling mines and the number of mines that each MOMAG must build. SUMIT then prints out separate mine transshipment schedules for each origin node that reports the following information: origin node, mode group, run number for the node-mode combination, destination node, total number of mines carried by the mode group, number of mode units needed, the time to start loading the mode group, estimated time of departure from the origin node, estimated time of arrival at the destination node, and the time in which the mode is available for loading for its next flight. It also warns the user if total supply is less than total demand for nodes recommended in the next run.

B. END TIME AND TIME PERIOD LENGTH

To find appropriate values for the end time and time period length for a new problem, the user should take the following steps: the user should initially execute GAMS using the relaxed version of ZOOM, which is faster since it allows binary variables assume any values between zero and one. To indicate the relaxed version of ZOOM, the user must change the problem identifier "MIP" to "RMIP" in the "SOLVE" statement, which is found on line 821 of the program (see Appendix A, page 55). The goal of making these quick runs is to identify the latest end time with an appropriate number of periods in which the solver reports an infeasible solution ("INFEASIBLE") in the section of the output labeled "SOLUTION REPORT." Then, the user should run SUMIT using an end time equal to the latest infeasible end time plus one extra time period with binary requirements enforced (replacing "RMIP" with "MIP" in the "SOLVE" statement). In most cases, adding an extra time period will make the solution feasible. If the problem is not feasible when the binary variables are enforced, the "SOLUTION REPORT" portion of the output will contain the phrase "INTEGER INFEASIBLE". If the solution is integer infeasible, the user should first verify that there is enough supply to meet the demand for the mine network output by the SUMIT. If the supply is sufficient, then the user should rerun the "MIP" version of ZOOM after increasing the end time by one extra time period until a feasible solution is found. Since the purpose of these final runs is to find the first integer feasible solution, the user may set the OPTCR at 1.0 to find the solution as quickly as possible.

C. OBTAINING NEARLY OPTIMAL SOLUTIONS

A general rule of thumb is recommended to obtain nearly optimal solutions for SUMIT. When the user makes the first run of the "MIP" version of ZOOM, the user should set OPTCR equal to 0.10 in line 16 of the program, rerun SUMIT, and verify that the reported solution is satisfactory. If the number of iterations is less than 10,000, which is given in the "SOLUTION REPORT" portion of the output indicated by "ITERATION COUNT", rerunning SUMIT with a very small OPTCR, such as 0.01, will probably yield an optimal solution within a reasonable number of iterations, but will not usually prove it. If the number of iterations is greater than 10,000, the user may rerun SUMIT at an OPTCR between 0.01 and 0.1 to find a solution that is optimal or close to optimal, but not proven. Proving optimality by using an OPTCR less than 0.01 is not recommended since this may result in an excessive solution time and excessive disk storage requirements.

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